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OF
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and Students in SCHOOLS, &c.
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THE
ELEMENTS
OF
Arithmetick.
IN
THREE BOOKS

Definitions.

1. **U**nity is that, by which every thing that is, is called one. *Unity is the beginning of every Number.*
2. Number is a Multitude composed of Units.
3. One Number is said to measure another, when the lesser being taken sometimes is equal to the greater.
As 4. measureth 12, because thrice 4. makes 12. Uni-ty measureth all Numbers.

The Elements

4. One Number is Multiplex of another, when the lesser measureth the greater; or when the greater containeth the lesser sometimes precisely.

5. The Aliquot part of a Number, is that which measureth the said Number. The Aliquant part which does not measure it.

The Number 2, is the Aliquot part of 10, because it measureth 10, being taken 5 times; but 3 is the Aliquant part of 10, because it does not measure 10; for being taken thrice it makes 9, and taken four times it makes 12.

6. Like Aliquot parts are those which measure their wholes equally; or which are equal times contained in their wholes.

2 and 3 are like Aliquot parts of the Numbers 10 and 15; because both 2 in 10, and 3 in 15 are contained five times; or both 2 measureth his whole 10, and 3 its whole 15, by the same Number 5.

7. Like Aliquant parts are those which are contained in their wholes, as often as one another; and in what Remains are contained equal times like Aliquot parts of them.

Or, like Aliquant parts are those which contain as many like Aliquot parts of their wholes as one another.

8 and 16 are like Aliquant parts of the Numbers 14 and 28; because that as 8 is contained once in 14 and 6 over, or thrice 2, that is 3 fourth parts of the 8; so 16 is contained once in 28, and 12 over, or thrice 4, which is also 3 fourth parts of 16.

Or thus, 8 and 16 are like Aliquant parts of the wholes 14 and 28; because that as 8 containeth four several parts of the whole 14, to wit 4 times 2, so 16 containeth four several parts of the whole 28, to wit four times 4.

8. In Numbers, the Ratio or Proportion is the mutual Habitude of two Numbers one to another, with respect to their excess, defect, or equality.

8. In



of Arithmetick.

3

In every Proportion there are two Terms, of which that is called the Antecedent which is in the first Place, and the other the Consequent; when the Antecedent is greater than the Consequent, it is called a Proportion of greater inequality, or of the greater to the lesser. When the Antecedent is less than the Consequent, it is called a Proportion of lesser inequality, or of the lesser to the greater. When the Antecedent is equal to the Consequent, it is called a Proportion of equality.

9. In Numbers, two Proportions are
 $A\ 8\ B\ 2$ equal the same, or alike, (they all signify one and the same thing) or four
 $C\ 16\ D\ 4$

Numbers A, B, C, D. are said to be Proportional when the lesser terms of each Proportion are contained alike in the greater; and they are contained alike, if the lesser (B, D,) are a similar Aliquot parts of the greater A, C, *b* or like Aliquant parts.

*a See def. 6.
 b See def. 7.*

We express equal Proportions thus; 8 is to 2, as 16 is to 4, or 8 has the same reason to 2, as 16 hath to 4, that is $8 \dots 2 :: 16 \dots 4$.

10. When there are many equal Ratios, and that the Consequent of the first is the Antecedent of the next (so as that the mean terms are taken twice) the Proportion is said to be continual, and the Numbers themselves are call'd continual Proportionals. *We express continual Proportions thus; 1 is to 2, as 2 is to 4, and as 4 is to 8, and as 8 is to 16, &c. That is $1 \dots 2 :: 2 \dots 4 :: 4 \dots 8 :: 8 \dots 16$.*

11. But if the Consequent of one of the equal Proportions is not the Antecedent of another, and so the mean terms are not taken twice, the Proportion will be discreet; and those Numbers are said to be

The Elements.

Proportional, without adding any other Word. We express discreet Proportions thus; 9 is to 3, as 12 is to 4; that is, $9 \dots 3 :: 12 \dots 4$.

12. When Numbers are continually Proportional, (A, B, C, D, E.) the reason of the first A to the third C, is said to be duplicate of the reason which the first A, hath to the second B; and the Ratio of the first A to the fourth D triplicate of the first A to the second B.

Of the Denominators and Composition of Ratios, see our Geometrical Elements, Book 5. Part 3.

13. A Number A, is said to multiply a Number B, when the Number B, to be multiplied is so often taken, as there are Unites in the Number A multiplying.

Or thus; The Number A multiplieth the Number B, when a Number C is found, containing the multiplicand B so often as the Multiplier A contains Units.

But Multiplication is more universally defined thus; The number A multiplieth the number B when a number C is found, which bears such Proportion to the Multiplicand B, as the Multiplier A hath to a Unit.

The Number C which is found, is called the Product or Fact; also when the Multiplier A is a whole Number, the Product C is always greater than the Multiplicand B, as is manifest from Def. 1. and 2. but when the Multiplier A is a Fraction, and less than a Unit, the Product C will be also less than the Multiplicand B; as is manifest from the last Definition.

Many numbers as 2, 4, 3. are said to be Multiplied one by another; if 2 by 4 makes 8, and the Product 8 be multiplied into 3, for the last Product 24 is that

that which is made by the Multiplication of the numbers 2, 4, 3.

It is the same thing with Arithmeticians to draw a Number into a Number, as to multiply a Number by a Number; as A by B, or A into B is all one.

14. A number A is said to divide a number B, when a number C is found, shewing how often the Divisor A is contained in B.

And so the Divisor A is a part of the Divided, or Dividend B, which is denominated from the Number found, (which therefore is called the Quotient.)

B 12	C 3
A 4	I

Or more Universally. A number A divideth the number B, when another number C is found, having such Proportion to a Unite, as the Dividend B hath to the Divisor A.

15. One number A is said to measure another B, by any number X (which is called the Quotient) when A, the number measuring being taken so often as there are Unities in the Quotient X is equal to the number measured.

B 16	X 8
A 2	I

This Definition differs from the preceding. 1. Because Division may be performed, although the Divisor doth not measure the Dividend. 2. And also the Divisor be greater than the Dividend, as shall be shewn in its place; therefore Division appears to be of a larger extent then Mensuration.

16. An even number is that which may be divided into two equal parts: Therefore some Number measureth every even Number by 2.

17. An odd number is that which cannot be divided into two equal parts, or that which differeth from an even number by an unity.

18. A number evenly even, is that which an even number measureth by an even number.

The Elements

Such is 24, which the even Number 6 measureth by the even Number 4.

19. A number evenly odd, is that which an even number measureth by an odd number.

Such is 12, which the even Number 4 measures by the odd Number 3,

20. A number oddly odd, is that which an odd number measureth by an odd number.

Such is 21, which an odd Number 7 measureth by an odd Number 3.

21. A prime or incompofit number is that which is measured only by a Unite, and consequently hath no Aliquot parts besides Unities.

Such are, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. and infinite others. Also every prime Number is necessarily odd, for otherwise 2 would measure it.

22. A compofit number is that which fome certain number besides a unite measureth; and consequently hath feveral Aliquot parts besides unites.

Such are, 4, 6, 8, 9, 10, 12, 14, 16. and infinite others.

23. Numbers prime the one to the other, are fuch, as no other common measure measureth but a unite.

The numbers 15 and 18 are faid to be prime the one to the other, becaufe that although fome numbers measure each, and fo neither is a prime number, yet no number measureth both; in like manner, 8, 10, 15, are three numbers Prime to one another, becaufe no number measureth them all three, and fo on.

24. Numbers compofit the one to the other, are they which fome number (besides a Unite) being a common measure, doth measure.

One of the given numbers may be the common measure.

The

of Arithmeticks.

The two numbers 8 and 10, are composi^t the one to the other, because 2 measureth both, likewise 5 and 15. are composi^t to one another. because 5 measureth both its self and 15; the three numbers 3, 9, 12. are composi^t to one another, because 3 measureth 9 and 12. and its self; 6, 10, 12, 16. are four numbers composi^t to one another, because 2 measureth all four.

25. A plain number is that, which is produced from the multiplication of two numbers: And the numbers which multiply one another, are call'd the sides of the said plain number.

Therefore every plain number is composi^t.

So 12 is a plain number, because it is made by the multiplication of 6 by 2, also 24 is a plain number, because it is made of 4 b. 6.

26. A Solid number is that which is produced from the multiplication of three numbers, and the numbers that multiply one another are called the sides of the said solid number.

Therefore every solid number is composi^t.

24 is a solid number, because it is made by the multiplication of three numbers 2, 3, 4. for $2 \times 6 = 6$ and $6 \times 4 = 24$.

27. Like plain and solid numbers are they, whose sides are proportional.

6, 24, are like plains; whose sides are 2, 3, and 4, 6, for $2 \dots 3 :: 4 \dots 6$. also 24. 92. are like solid, because the sides of one, 2, 4, 3, are proportionable to the sides of the other, 4, 8, 6.

28. A square number is that which is made by the multiplication of two equal numbers, or by the multiplication of any number by its self, which is called a square root.

The first square is 4, which is made of 2 by 2 or of 2 multiplied into its self; the second is 9, which is made of 3 in its self, and so on in infinitum.

29. A Cupe is that which is made by the multiplication of three equal Numbers or of the same number thrice taken, which is called a Cube root.

The Elements

The first Cube is 8 which is made by the Multiplication of 2 thrice taken, 2, 2, 2. for $2 \times 2 = 4$ and $4 \times 2 = 8$ the second is 27 which is made by the multiplication of 3 thrice taken, 3, 3, 3; for $3 \times 3 = 9$ and $9 \times 3 = 27$.

10. A perfect Number is that which is equal to all its aliquot parts.

The first perfect number is 6 for all the aliquot parts thereof are 1, 2, 3, which together make 6. The Second 28. for all its aliquot parts are, 1, 2, 4, 7, 14. which together make 28, of these see the Last Prop. Book 9, and its Scholium.

A X I O M S.

A. B. 1. **N**umbers, A, B, which are equimultiples of the same number Z are equal, and numbers which are equimultiples of equal Numbers are equal.

2. Numbers A, B, whose Equimultiplex is the same Number are equal, and those Numbers are equal whose equimultiples are equal.

3. Those Numbers are equal which are the same part of the same Number, as a half, a third, or a fourth, &c.

And those Numbers are equal, which are the same part of equal Numbers.

4. Those Numbers are equal, of which one Number is the same part, and those Numbers are equal of which equal Numbers are the same part.

5. Unity measures every Number by the Units that are in it, that is by the same Number.

6. Every Number measureth its self by a Unite.

A 4. B 3. C 7. If one Number A, multiplying another B generateth any Number C, the multiplied B measureth the generated C, by the multiplier A.

This is manifest from the 13. and 15. Defin.

8. If a Number A measureth or divideth another number C, by the quotient B, also the quotient B multiplying

Book of Arithmetick

multiplying the measuring A, produces the measured C, or the measuring A multiplied by the quotient shall produce the measured C.

This is manifest from Defin: 13. 15. 14.

9. That a greater than any Number may be taken

10. If a Number A measures how many Numbers (ever BC, CD, DE, it also measureth, the Number composed of them B E.

A —
B — C — D — E

11. If a Number A measureth any Number B, it also measureth every Number C, which the said Number measureth.

A — 3
B — 4
C — 12

12. The Number A that measures the whole B C, and the part taken away BD doth also measure the residue DC,

A —
B — D — C

Arithmetick

to the seventh of

EUCLID

THE

PROP. I.

THE
FIRST BOOK
OF THE
ELEMENTS
OF

Arithmetick,
And the Seventh of
EUCLID.

In the Citations of the Book, Euclid's number is retained.

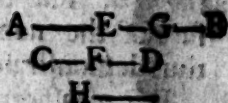
PROP. I.

IF from two unequal Numbers the lesser be continually taken from the greater by an alternate Subtraction; and the remainder do never measure the preceeding, until unity be come to: then are the given numbers (a) prime the one to the other.

(a) See
def. 14.

Let

Let the two Numbers, AB and CD be given; now CD being taken from AB so often as it can leaveth EB , and EB being taken so often as it can from CD , leaveth FD , and FD being taken so often as it can from BE leaveth a Unity. I say that the Numbers AB , CD , are Prime the one to the other.



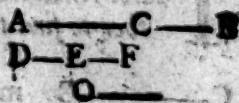
If it can be, let the Numbers AB , CD , have a common measure H , therefore because you will have H , to measure CD , and CD measures (b) AE , also (c) H shall measure AE ; but you will have H also to measure the whole AB , therefore also (d) it measureth the remainder EB , but EB measureth (e) CF , therefore also H (f) measureth CF ; wherefore seeing you would have H , to measure the whole CD , it will also measure the remainder (g) FD ; But FD measureth (h) EG , therefore also H measureth (i) EG ; but I have shewn above that H also measureth the whole EB , therefore also it measureth the remainder (k) GB , a number a unite which is absurd.

(b) Hyp.
 (c) Ax. 14.
 (d) Ax. 12.
 (e) Hyp.
 (f) Ax. 11.
 (g) Ax. 12.
 (h) Hyp.
 (i) Ax. 11.
 (k) Ax. 13.

PROP. II.

TWO Numbers (AB, DF) given, not Prime the one to the other, to find their greatest common Measure.

The lesser DF , taken from the greater AB so often as it can leaveth CB , CB taken so often as it can from DF leaveth EF and so on.



By this alternate Subtraction at length is left some number which measureth the foregoing, for if we came to Unity by Subtraction, the given numbers would be Prime contrary to the Hyp: Therefore let the remainder be EF , and let it measure the preceding

(a) Preced.

ing

ing CB, I say that EF is the greatest common measure of the numbers AB, and DF, which shall be thus demonstrated:

- (b) *Constr.* EF measureth (b) CB, and CB measureth (c) DE, therefore also EF measureth (d) DE, but EF also measureth its self, therefore EF (e) the whole DF; but DF, measureth (f) AC, therefore also EF, measureth (g) AC, but EF also measureth (h) CB, therefore also EF measureth the whole (i) AB, therefore EF is the common measure of AB, and DF.
- (j) *Constr.* And that it is the greatest is thus shewn; If it can be let there be another O, greater than EF, because you will have O to measure DF; and that DF measures AC, also O, shall measure (k) AC; But you will have O also to measure AB, therefore also O shall measure (l) CB, but CB (m) measureth DE, therefore also O measureth (n) DE, wherefore seeing you would have O to measure the whole DF, it also measureth the (o) remainder EF, lesser then its self which is absurd.

Corollary.

A Number (O) that measureth two Numbers (A B, DF) also measureth their greatest common measure (EF)

PROP. III.

T Hree Numbers given (C, D, E) not Prime to one another to find their greatest common measure.

(a) *Prop.*

Find O the greatest (a) common measure of the two Numbers given C, D. If O also measures E; it will be the greatest common measure of the three Numbers C, D, E; for if another S were greater then O; then also S, would measure O by the precedent Coroll. which is absurd, seeing S. is put greater than O. But if O does

does not measure E, yet O and E will be composited the one to another:

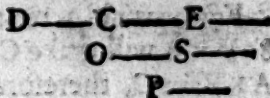
For seeing C, D, E (b) are three numbers composed to one another, some common measure will (c) measure them, and also the said (d) O: therefore I say that P. the greatest common measure found of the two numbers O, E is the greatest common measure of the three Numbers C, D, E. (b) Hyp.
(c) De. 24.
(d) Cor. prec

For seeing P (e) measureth O, and O measureth (f) C, D, also P (g) measureth C, D; but (h) P. also measureth E, therefore P. measures the three C, D, E; and that it is the greatest common measure 'tis thus shewn; let another S. if it can, be greater than P, therefore because S. measureth C, D, E, it will also measure (i) G the greatest common measure of C, D: Wherefore because S measureth E and O; S will again (k) measure their greatest common measure (l) P, the greater the lesser, which is absurd. (e) Constr.
(f) Constr.
(g) Axi. 11.
(h) Constr.
(i) Corol.
(k) Corol.
(l) Constr.

Corollarys.

1. After the same manner may be found the greatest common measure of four, or more Numbers not Prime the one to the other.

2. The Number S. measuring any Numbers E, D, C, also measureth their greatest common measure P, which is manifest from the last part of the preceding Demonstration.



PROP. IV.

THE Fourth and rest of the Propositions to the 15. inclusive are contained in the Universal Propositions of the Fifth Book of our Elements of Geometry.

PROP.

PROP. XVI.

IF two Numbers (A, B) multiplying themselves mutually the one by the other, produce the Numbers C, D , the Numbers produced are equal to one another.

Because A multiplying B maketh D , it will be (a) $1 \dots A :: B :: D$ therefore by permutation (b) $1 \dots B :: A :: D$, again, because B multiplying A , makes C , it will be (c) $1 \dots B :: A :: C$, therefore $A \dots D :: A \dots C$ therefore also (d) $C = D$. W. W. Dem.

(a) Def.

13. 7.

(b) 16. 5. A 6

(c) Def.

14. 7.

Unky 1

C 24

B 4

D 24

Corollary.

A. B.
C.

1. **I**F a Number A , multiplying another number C , produces a third C . the multiplier A , will measure the Product C . by the Multiplicand B . for because $A \times B = C$, also $B \times A = C$ by this Prop. wherefore seeing A which at first was the multiplier is now the multiplicand, in respect of the same Product C , it is manifest from Ax. 7. that A measureth C by B .

2. If A measureth or divideth C by B , also the quotient B shall measure the same C . by A .

For because A measureth C . by B , therefore by Ax.

8. $B \times A$ will make C , wherefore by this 16 Prop. also $A \times B = C$, therefore by Ax. 7. B measures C by A . W. W. Proposed.

PROP. 17.

IF a number (A) multiplying, Numbers (B, C), how many soever, produce so many other Numbers AB, AC , the Numbers produced of them shall be proportional to the Numbers multiplied.

For seeing $A \times B = AB$, it will be (a) (a) def.

Unity

1. $A :: B :: AB$, again seeing $A \times C = AC$.

I

AC it will be (b) 1. $A :: C :: AC$ (b) *ibid.*

A

therefore (c) $B :: AB :: C :: AC$ (c) 11. 5.

B. C.

therefore by permutation $B :: C :: (d) (d) 16. 3.$

$AB. AC. AB. AC. W. W. Demonstrated.$

SCHOLIUM.

We have here begun to express the product of Multiplication, only by the mutual apposition of the Multiplying Numbers; of which see what has been wrote before the beginning of this Seventh Book: if this method is used (which for the most part will be wonderful compendious) the 17. Proposition may be more commodiously express after this manner.

Numbers how many soever $A B, B C$ having one common side B have the same proportion the one to the other as the other sides A and C have.

The sides of a number are they; by the Multiplication whereof the said number is produced; moreover that tyoes may the better use themselves to this method, Which (as I said before) is very commodious and compendious, besides the Example brought in this Proposition; I think meet to add some other Examples.

1. $AA .. AB :: A .. B$
2. $AB .. BB :: A .. B$
3. $AAA .. AAB :: A .. B$
4. $ABB .. BBB :: A .. B$

In the 1 the common side is A , and there are left the second A , and B ; In the 2. the common side is B , and there remains A and B . In the 4, the common side is BB and the remains are A and B .

5. A, AB, BB are in continual proportional in the reason of A to B which is manifest from 1 and 2.

6. AA, AAB, ABB, BBB, and in continual proportionals in the reason of A to B, as is manifest from 3 and 4.

9. AB, AC, AD, AE, AF, have the same proportion the one to the other, as B, C, D, E, F have, which is manifest from the Proposition.

What great use there is of this Scholium, for the ease expediting of prolix and difficult Demonstrations, will appear both in these Books, and in Practical Arithmetick.

PROP. VIII.

IF Numbers B, C, how many soever multiplying the same Number A, produce so many other Numbers D, F, the numbers produced D, F are proportional to the Multiplying numbers B, C.

For seeing $B \times A = D$ also (a)

B 2. C 4. $A \times B = D$; and seeing $C \times A = F$

A 3. also (b) $A \times C = F$, therefore (c)

D 6. F 12. $B :: C :: D :: F$, W. W. Dem.

Corollary.

A Number A dividing or measuring any numbers, J. P. produceth Quotients (R, S) proportional to the numbers divided or measured.

For because A, measureth or divideth the said J. P. by the Quotients R. S it is manifest by the 8th. axiom that $A \times R = I$ and $A \times S = P$. therefore by the 17 Prop. $I :: P :: R :: S$.

Other ways.

(d) def. 14. I 12. P 26.

(e) 16. 5. A 4

(f) def. 14. R 3. S 5.

Unit

E

When A measuring or dividing A maketh R. it will be (d) $I :: R :: A$. I therefore (e) by permutation $I :: A :: R$. I. again seeing A measuring P makes S, it will be (f) $I :: S :: A$. P. therefore by permutation $I :: A :: S$. P wherefore seeing I have already ready

ready shewn that also $R : I :: I : A$ it will be $(g) (g) 11. 5.$
 $R : I :: A : P$ and by permutation $R : A :: I : P$
 which was to be Demonstrated.

PROP. XIX.

IF four Numbers A, B, C, D , are proportional; the number A & D produced of the first A , and fourth D will be equal to the number produced of the second B and third C , and the contrary.

1 Part. $A \times C = AC, AC \dots AD :: A \times D = AD, AD \dots BC$ (a) 18. 6.
 $(a) C \dots D$ and $AC \dots BC :: (b) A \dots B$ (b) by the same.
 B wherefore seeing $(c) A \dots B :: C \dots D$ $EC 12. AD 12.$ (c) Hyp.
 D also $(d) AC \dots AD :: AC \dots BC$ $AC.$ (d) 11. 5.
 therefore $(e) AD = BC$ W. W. Demon. (e) 9. 5.
2 Part because by hypothesis $AD = BC$ it will be $(f) 7. 5.$
 $(f) AC \dots BC :: AC \dots BC$ but $AC \dots BC$ (g) $A \dots B$ (g) 11. 5.
 and $AC \dots AD :: C \dots D$ therefore $(h) A \dots B :: C \dots D$ (h) 11. 5.
 W. W. Demon.

Corollary 1.

IF two numbers B, C measure or divide the same Number A , by the quotients O, P , it will be $B \dots C :: P \dots O$.

For because B, C measures or divides A by O and P , therefore by ar. 8. $B \times O = A$ and $P \times C = A$ that is $B \times O = P \times C$
 Therefore by this Proposition $B \dots C :: P \dots O$

Corollary 2.

IF the ratio $A \dots B$ be greater than $C \dots D$; then $A \times D$ produced of $A \times D$ shall be greater than $B \times C$ produced of $B \times C$ and the contrary.

Part 1 $A \times C = AC, AC \dots BC$ (a) Sch.
 $:: (b) A \dots B$ that is greater (b) $A \times D = AD, AD \dots BC$ (b) 4. 4.
 than $C \dots D$ that is again (c) $C \times D = CD, CD \dots BC$ (c) 4. 4.
 greater then $AC \dots AD$, therefore $AD \dots BC$ (c) Sch.
 BC is lesser (d) then AD . (d) pro. 17.
 Part (d) 19. 5.

Part 2. Because AD is already agreed to be greater than BC the ratio AC : BC shall be greater then the ratio (e) AC : AD, but AC : BC :: (f) A : B and AC : AD :: (g) C : D, therefore also the ratio A : B is greater then that of C : D.

PROP. XX.

If three numbers (A B C) are proportional, the number (A C) produced by the extremes is equal to the square (B B) of the mean.
 And if the number contained under the extremes be equal to the square of the mean, the three numbers are proportional.
 Part 1. Write the mean B twice, therefore by hyp. A : B :: B : C seeing now four proportionals are had, the number A generated from the extremes AC, will be equal to the number generated from the second B and third B, that is to the square of the said B.

Part 2. Is Demonstrated after the same manner, from Part the second of the foregoing.

PROP. XXI.

Numbers (A and B) being the least of all in their proportion do equally measure the Numbers C, D having the same proportion with them.

For seeing by hyp. A : B :: C : D, also by permutation A : C :: B : D, therefore A and B are like aliquot or aliquant parts of C and D, (and that A B are less then C, D, is manifest from the hyp) but that they are not like aliquant parts I thus shew if A, B are like aliquant parts of the said C, D then A containeth O such an aliquot part of the said C, as B containeth of the said D, suppose P, and consequently C : O :: B : P, D and by

permutation $d \ O \ . \ . \ P \ . \ . \ C \ . \ . \ D$ that is $A \ . \ . \ d \ 16. 9.$
B but A, B , contain the said O, P . therefore
 A, B , are not the least of all that have the same
 proportion, which is against the hyp. Therefore A, B
 are not like aliquant parts of the said C, D . it remains
 therefore that they are like aliquot parts, and conse-
 quently do equally measure the said C, D . *W. W.*
Demon.

Corollary.

VIXX 2099

After the same manner it will be A, B, C, D, E ,
 Demonstrated that Numbers O, P, Q, R, S ,
 A, B, C, D, E how many soever F, G, H, I, K ,
 being the least of all that have the
 same proportion with them do equally measure so
 many numbers in the same proportion with them.

PROP. XXII.

This is contained in Prop. 22. Book 5.

PROP. XXIII.

Numbers a prime the one to the other $A \ 15. \ B \ 8. \ a \ def. \ 23.$
 A, B , are the least of all that have
 the same proportion with them $C \ 3. \ D \ 4.$
 If it be possible, let others C, D : *Unity.*
 be the least and proportional to the
 said A, B , therefore C, D , measure A, B equally $b \ 21. 7.$
 that is by the same number E ; Wherefore because C *def. 23,*
 measures A by E ; $1. \ . \ . \ R \ . \ . \ C \ . \ . \ A$, and by permuta-
 tion $1. \ . \ . \ C \ . \ . \ E \ . \ . \ A$, but *Unity* measures C . therefore
 also E measures A , after the same manner it may be
 shewn that E measures B seeing therefore that E mea-
 sures both A and B , they are not prime the one to the
 other contrary to the hypothesis.

D 2

Corol.

Corollary.

AFTER the same manner it may be demonstrated that if Numbers how many soever A, B, C, D, E , are Prime one to the other, they are the least of all Numbers, having the same proportion with them.

PROP. XXIV.

Numbers A, B , being the least of all that have the same proportion with them, are Prime the one to the other.

If they are not Prime, let the common measure E , measure A , by C , and B , by A , whence $A \dots B :: C \dots D$, wherefore seeing C, D , are less then A, B , A, B are not the least of all in the same proportion with them, which is contrary to the Hypothesis.

Corollary.

AFTER the same manner it may be demonstrated, that if Numbers how many soever A, B, C, D , are the least in their proportion they are also Prime the one to the other.

PROP. XXV.

IF a Number (N) measures one (A) of two Numbers (A, P) being Prime the one to the other, it shall be Prime to the other Number (B).

For

For if N, B, are not Prime the one to the other, let X. measure both, wherefore be- $A, B,$
 cause X. measureth N, and N measureth $N, X, a \text{ Hyp.}$
 $A,$ therefore also X. will measure $b A,$ but $b \text{ ax. 11.}$
 you would have X, to measure B, therefore $A, B,$
 are not Prime the one to the other, contrary to the
 Hypothesis.

PROP. XXVI.

IF two Numbers ($A, B,$) are Prime to any Number ($C,$) the
 Number (AB) produced of them shall be Prime to the
 same ($C.$)

For if $A, B,$ and $C,$ are not Prime
 the one to the other, let D measure $A. 7. B. 3.$
 both by F, therefore $D \times F, a = A C 8. a \text{ ax. 8.}$
 B but also $A \times B b = AB,$ therefore $A. B. 21. b \text{ Hyp.}$
 $D. A :: C B. F,$ now because A and $D \rightarrow F \rightarrow c 19. 7.$
 $C.$ are Prime the one to the other,
 and that you would have D to measure C, therefore d
 D will be Prime to A, wherefore D and A, are the $d \text{ prac.}$
 three at least in their proportion, and therefore they $e 23. 7.$
 equally measure $f B F.$ in the same proportion with $f 21. 7.$
 them, to wit, D measures B; and A the said F, and
 because you would have D, also, to measure C, there-
 fore D shall measure both C, and B, and therefore C,
 and B, are not Prime the one to the other, contrary to
 the Hypothesis, therefore AB, shall be Prime to C. W.
 W. demonstrated.

PROP. XXVII.

IF two Numbers ($A, B,$) are Prime the one to the other,
 also the Square (AA) of one of them shall be Prime
 to the other ($B.$)

Write A twice, now because the first $A 4. B 7.$
 A is Prime to B. also the last A, shall $A A 16. a \text{ Hyp.}$
 be Prime to B; therefore the fact of A $A.$
 $\times A,$ that is, $b AA.$ is Prime to B. $b \text{ prac.}$

The Elements

PROP. XXVIII.

IF two numbers (A B) are prime to two numbers (C D) each to either of both; the numbers also produced of them (A B, C D) shall be prime to one another.

- (a) Hyp. For because (a) A and B are prime to
 (b) 26. 7. A. B. C, also (b) A B shall be prime to C: again because A and B are prime to (c) D,
 (c) Hyp. AB. D. also AB shall be prime to D; seeing
 (d) 26. 7. C. CD. therefore C and D are prime to AB, also CD, the Number produced of them, shall be (d) prime to AB. W. W. Demonstr.

PROP. XXIX.

IF two Numbers (A, B) are prime the one to the other, their Squares AA, BB, and Cubes, AAA, BBB, and so on, shall also be prime the one to the other.

- Because A B are prime the one to the other; also AA is (a) Prime to B. and because AA, and B are prime the one to the other; also (b) AAAA BBBB BB shall be prime to AA, which was to be first Demonstrated.
 (a) 27. 7. A A B B
 (b) 27. 7. AA BB
 AAA BBB
 AAAA BBBB
 (c) 27. 7. Again because A, B are prime to one another, also BB will be prime to (c) A; therefore B and BB are prime to A. It is also already shewn, that A and AA are prime to B; wherefore AAA, produced of A x AA is prime to (d) BBB, produced of B x BB, which was the other.

- Also because A and AA are prime to B, also AAA produced of them is prime to (e) B; after the same manner, because B x BB are prime to A, also BBB shall be prime to (f) A; now because A and AAA is prime to B, and B and BBB also prime to A, the

prop.

products AAAA, and BBBB, produced of them shall also be (g) prime to one another; and so on infinitely.

(g) 28. 7.

PROP. XXX.

IF two Numbers A, B, are prime the one to the other; also both together $A + B$ shall be prime to either of them.

And if two numbers together $(A + B)$ be prime to any one of them; also the numbers given shall be Prime the one to the other.

Part. 1. For if $A + B$, is not prime to A, let the Number C measure them, which also must (a) measure B; and then A, B, $A + B$ are not prime the one to the other, $C \overline{A+B}$ which is against the Hypothesis.

(a) Ax. 11.

Part. 2. If A, B, are not prime to one another, let C measure them, which consequently (b) will measure $A + B$, contrary to the Hypothesis.

(b) Ax. 12.

Corollary.

IF $A + B$ be prime to A, also $A + B$ shall be prime to B; for because $A + B$ is prime to A, A and B shall be prime the one to the other, by Part. 1. then because A and B are prime to one another, also by part. 1. $A + B$ shall be prime to B.

PROP. XXXI.

EVery prime Number (A) is prime to every other Number B, that it does not measure.

For if A and B are not prime to one another, some Number C different from A, B, will measure them; but A, by Hypothesis doth not measure B; therefore (a) C is not a prime Number.

A. B.
C.

(a) Def. 21

PROP. XXXII.

(a) Def. 25 IF a prime Number C, measureth a plain (a) Number A B, it shall also measure one of the sides of the plain Number.

Let the prime Number C measure the plain Number A B by D; if now C does not measure A, then C, A (b) shall be Prime to one another and therefore the (c) least in the their Proportion; but because C measureth AB by D, therefore $C \times D = (d)$ AB; but also $A \times B = AB$, wherefore $C : A :: (e) B : D$. Wherefore seeing C, A are the least in the same Proportion, C will measure (f) B; after the same manner it will be proved that C measures A, if it does not measure B. Therefore the Proposed is manifest.

PROP. XXXIII.

Some prime Number measureth every Composi Number.

Let any composi Number F be given; now one or more Numbers, whereof let the least be N (a) measures it. I say that N is a prime, for if it is not, some other P will measure it, which then will also measure F. (b) F, although it be less then N, which is absurd; seeing N is put for the least of all those that measure F.

PROP. XXXIV.

Every Number is either a prime, or may be measured by some prime.

This is manifest from the preceeding Prop.

PROP. XXXV.

Numbers (*A, B, C*) how many soever being given, to find the least in the same Proportion with them.

Find the greatest common measure (*a*) *O* of the (*a*) *3, 7*, given Numbers *A, B, C*, which dividing or measuring them will quote or make *A. B. C.*
D, E, F; I say that these are the least *O —*
 in the same Proportion with the given *D, E, F*
 Number *A, B, C*; if they want a com- *P, Q, R*
 mon measure, they are prime the one *X —*
 to the other, by Th. 7. and so the least
 in their Proportion by 23. 7 : But that *D, E, F*, are
 Proportional is manifest from the Coroll. to Prop. 18.
 7; and that they are the least I thus shew; If it be
 possible let others *P, Q, R*, be the least in the same
 Proportion with *A, B, C*, given, therefore *P, Q, R*,
 measure (*b*) *A, B, C* by the same Number, as by *X*. (*b*) 21. cor.
 Therefore *P x X = (c) A*, and also *D x O = (d) A*, 7.
 wherefore (*e*) *P .. D :: O .. X*; but you would have *P* (*c*) Ax. 8.
 to be less, than *D*, therefore also *O* is less then *X*, (*d*) Constr.
 now because *P, Q, R* measure *A, B, C* by *X*, it is ma- & Ax. 8.
 nifest that *P, Q, R* multiplied by *X* (*f*) produces *A, B, C*, (*e*) 19. 7.
C; and consequently that *X* measures (*g*) *A, B, C* by (*f*) Ax. 8.
P, Q, R; Therefore *O* is not the greatest common (*g*) Ax. 7,
 measure of the said *A, B, C* given, which is against
 the Hypothesis.

Corollarys.

THE greatest common Measure of Numbers how many soever, measureth them by the least of all the numbers, which are in the same proportion with them.

PROP.

PROP. XXXVI.

TWO Numbers (A, B,) being given to find the least number that they do measure.

A. B. If the given numbers A, B are prime to one another, the number AB produced of them, is that sought; for because $A \times B = AB$, A shall measure A, B, and because $B \times A = AB$, B shall measure AB,

a ax. 7.
b 16. 7.

N. P. therefore both A and B measure AB, but that AB is the least number that A and B measure, I thus shew; If it be possible let O be less, and let A and B measure the said O by N and P, wherefore $A : B :: P : N$, but because A, B, are prime numbers, they are the least in their proportion, and consequently A measures P, and B measures N; Now because $A \times B = AB$ and $A \times N = O$ as is shewn above, it will be $B : N :: A B : O$, but I have already shewn that B measures N, therefore also AB will measure O, lesser than its self which is absurd.

c cor. 19. 7.
d Hyp.
e 23. 7.
f 21. 7.
g 17. 7.

A. B. If the given Numbers A, B are not prime to one another, find C, D the least in the same proportion with them, then $A \times D = AD$ or $B \times C = BC$ I say that AD or BC (for they are equal) is the least which

b nec.

j 19. 7.

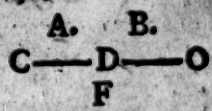
A, B measure; for if you would have A and B to measure O, some other less than AD. I will shew by the like reasoning as above that AD will then measure O, less than its self.

PROP. XXXVII.

IF two Numbers (A and B) measure any other Number (CO) also (F) the least which the said (A, B,) measure, shall measure the same (CO.)

For

For if F doth not measure C, O, it being taken from C O, so often as it can, leaves D, O, less then its self, but ^a because both A and B measure F, and that F measures C D, also ^b A and B shall measure the said C D; but A, B do also ^c measure the whole C O, therefore also they shall measure the remainder ^d D O less then F, which is against the Hypothesis.



^a Hyp.
^b ax. II.
^c Hyp.
^d ax. 12.

PROP. XXXVIII.

THREE Numbers (A, B, C,) being given to find the least number which they measure.

Find the ^a least P which A and B, measure, if the third C, doth also measure P, then P, will be the least, that A, B, C, measure; for if it be possible let there be another R, less; that A, B, C, measures; therefore P, is not the least that A, B, measure, which is against the Hypothesis.

A, B, C,
P, R,
X.

^a 36. 7.

But if C does not measure P, find ^b R the least that P and C measure, and R shall be the least, that A, B, C measure, for because, A, B measure P, and P measures R, also A and B shall measure ^c R. but C also measures the said R, therefore the three Numbers A B C measure R.

^b by the same.
^c by the prac.

And that R, is also the least is thus shewn, if it be possible let X, be less than R, which A, B, C, measure, therefore P the least that A, B measure, shall also measure ^d X, therefore seeing C, P, measure X (C. by the Hyp. and P. by what is already demonstrated) a number less than R; R shall not be the least which C, P, measure, which is absurd and against the Construction.

^d By the prac.

Cor.

Corollary.

1. **B**Y the same artifice, if Numbers how many soever are given, the least which they all measure, may be found.

2. If three Numbers A, B, C, (or more) measure any other number X, also R, the least which A, B, C, measure, shall measure X.

a Hyp.

b 27. 7.

The Construction being made as above, seeing A, B, a measure R; also P b shall measure X, and seeing C, P, measure X. (C by Hyp. and R, by what is already demonstrated) also R shall measure X. W. W. demonstrated.

PROP. XXXIX.

IF a number (A,) measureth another number (B,) by the quotient (C,) the quotient shall be a part of the measured (B,) denominated from the measuring (A.)

a 12. 9.

A. 3.

B. 24.

(C 4

For seeing A measureth B by C, also C shall measure a B by A, (that is C being so often taken as there are Units in A,) maketh B, and so C, is a part of B. denominated from A.

PROP. XL.

IF a Number (A,) hath any part whatsoever (B,) the Number (E) denominating the part, shall measure the same.

A 24.

B. 6.

E. 4.

Because B. is a part of the Number A denominated from E therefore E measures A by E, and therefore again E, measureth b A by B.

b 11. 9.

PROP.

PROP. XII

TO find the least Number which hath or containeth the parts denominated from the Numbers A, B, C. given.

Find a G the least which the given denominators A, B, C. of A. 2. B. 3. C. 4. the parts, measure, I say that G. 12. G is the number sought.

a 38. 7.

For A, B, C, measure the said G. by the Quotients P, Q, R, therefore P, Q, R, are $\frac{1}{2}$ parts of the said G. denominated by A, B, C; but that G. is the least is manifest; for if X less then G, hath parts denominated from the Numbers A, B, C; A, B, C, c shall measure the said X, and therefore G would not be the least which they measure, contrary to the Hypothesis.

P. 6 Q. 4. R. 4. X.

b 39. 7.

c prec.

Corollary.

THE least number which numbers A, B, C, how many soever given measure, is also the least of all that have parts denominated from the said given numbers.

PROP. I.

THE

least number which numbers A, B, C, how many soever given measure, is also the least of all that have parts denominated from the said given numbers.

THE
SECOND BOOK
OF THE
ELEMENTS
OF
Arithmetick,
And the Eighth of
EUCLID.

PROP. I.

IF there be divers Numbers, how many soever (A, B, C, D) whose extrems A, D , are prime the one to the other, then those Numbers A, B, C, D , are the least of all in the same Proportion with them.

For if it be possible let there be as A, B, C, D . many others E, F, G, H . less and E, F, G, H . Proportional to the said A, B, C, D .

there-

therefore by Equality of Reason (a) $A : D :: E : H$.
 And because A and D prime one to the other, they
 are the (b) least in the same Proportion with them;
 and consequently will measure the said (c) E. H lesser
 then themselves, which is absurd.

Euclid makes the Numbers A, B, C, D, to be in continual Proportion, which does not appear by the Demonstration to be requisite.

PROP. II.

To find out the least Numbers continually Proportional, as many as shall be required, in the Proportion given of A to B.

Let A, B, be the least Terms of the given Ratio,

now $A \times A = AA$, $A \times B = AB$, $B \times B = BB$.

then AA, AB, BB, are the three least Terms in the Proportion of A to B.

Unity.
 $A \ 2 \ B \ 3$
 $AA \ AB \ BB$
 $A \ 4 \ B \ 9$
 $AAA \ AAB \ ABB \ BBB$
 $8 \ 12 \ 18 \ 27$

That they are in the same Proportion as A to B is manifest from Prop. 17. 7. and its Scholium. And that they are the least I thus shew; because A, B, are the least in their Proportion, they are prime the one to the other; wherefore also their Squares AA, and BB, are prime to one another; therefore AA, AB, BB, are the three least Terms in their Proportion; that is in the Ratio of A to B.

Then A multiplying the three Numbers already found makes AAA, AAB, ABB, and B multiplying the third BB, makes BBB, these four are the least Proportionals in the Ratio of A to B; But that they are Proportionals in the Reason of A to B, appears from 17 E. 7. and its Scholium; and that they are the least is manifest from 29 E. 7. and from what goes before.

For

(e) 264. For AAA, BBB, are (e) prime the one to the other, are consequently AAA, AAB, ABB, BBB, are the four (f) least in their Proportion, which is as A to B. In the same manner may be found four, five six, continual Proportionals, and so on in infinitum, the their Proportion.

Corollarys.

1. Hence if three Numbers AA, AB, BB, being the least are continually Proportional, their extremes are Squares, if four, their extremes are Cubes and so on.

2. The Extrems of Numbers found to be the least, and in continual Proportion, are prime the one to the other; this is manifest from their Genesis and from 292 and 7.

3. Two Numbers, A, B, being the least of a given Ratio, measure all the other Terms, altho they are infinite, for seeing A, B, generate all the rest; they will also measure them by Axiom 7. and Corol. Prop. 16. E. 7.

4. Unity, A, AA, AAA, are continually Proportional, or continual Proportionals; likewise also, Unity, B, BB, BBB are, for seeing $A \times A = AA$, it will be (g) $1 \dots A \dots A \dots AA$, and seeing $A \times AA = AAA$, it will be (h) $1 \dots A \dots AA \dots AAA$, therefore A, AA, AAA, are continually Proportional. (g) Def. 13. (h) by the same.

5. Between the Extrems AAA and BBB, there fall as many means as between them and Unity, which is manifest from what is already Demonstrated.

PROP. III.

If there are continual Proportionals, A, B, C, D, how many soever, being also the least of all that have the same Proportion with them, their Extrems A, D, are prime to one another.

a Find

* Find two Numbers E, F, which are the least in the Ratio of A to B, then by the preceding find so many, O, P, Q, R, the least in the Ratio of E to F; therefore because both O, P, Q, R. as well as A, B, C, D. are the least in the Ratio of E to F, and as many as one another; they must of necessity be one and the same Numbers. But the Extreams O, R, are *b* prime to one another; therefore also A, D are prime to one another. W. W. Demonstrated. *b* Cor. 2: *prac.*

Hence a Series of continual Proportionals being in their least Terms, and whose extreams are consequently prime the one to the other, cannot be continued further, as shall be demonstrated in Prop: 17. 9.

PROP. IV.

Proportions or Ratios, how many soever being given in their least Numbers, to continue them also in the least Numbers.

1. Let there be given two Ratio's, A to B, and C to D, in their least Terms; find *c* O the least Number which B and C measure; then so often as B measures O, so often let A measure the Number N; and so often as C measures O, let D measure P; I say that N, O, P, are the least that continue the given Ratio's, A to B, and C to D; and that they do continue the given Ratio's is manifest from their Genesis, by reason whereof $A \dots N :: B \dots O$, and $C \dots O :: D \dots P$ wherefore by permutation $A \dots B :: N \dots O$, and $C \dots D :: O \dots P$; and that they are the least, I thus shew; For if it be possible let Q, R, S, lesser than the former continue the given Ratio's; wherefore because $A \dots B :: Q \dots R$, and that A, B are the least in their Reason, it is manifest that A, B, measure the said Q, R, for the same Reason C, D will measure R, S; wherefore because both B and C measure R, also O, the least *d* 21. 7. } which

37. 7. which B and C measure, e shall measure the said R; the greater the lesser, which is absurd.

2. Let there be given three Ratios in their least Terms, A to B, C to D, E to F, find O the least, which B and C measure; and so often as B, C measure O, so often let A, D, measure the Numbers N P; if then also E measures P, as often as F measures Q. I say, that N, O, P, Q are the least that continue the given Ratios or Proportions.

But that N..O::A..B, and O..P::C..D, and P..Q::E..F, appears from the construction its self; and that they are the least I thus shew; If it be possible let others lesser R, S, T, V, continue the given Ratios. I will shew as above that the greater O measures the lesser S, which is absurd.

But if E does not measure P, find the least S, which P and E measure, and so often as P measures S, so often let O, N, measure the Numbers R, Q; also so often as E measures S, so often let F measure T: I say, that Q, R, S, T are the least which continue the given Ratios.

For by Construction it is manifest that S..T::E..F, also that Q, R, S are proportional to the said N, O, P; but N, O, P, continue (as is shewn above) the Ratios A..B and C..D; therefore also Q, R, S continue the same, and consequently seeing that E..F::S..T, it is manifest that Q, R, S, T continue the three Reasons A..B; C..D; E..F; but that Q, R, S, T are the least, I thus shew; if it be possible let others less, V, X, Y, Z continue the three Ratios given, wherefore because A..B::V..X, and that A, B are the least in their Proportion, B will measure X, after the same manner it may be shewn that C measures X, therefore also N O the least which B and

o Hyp.

f 21. 7.

n 37. 7.

and C measure, will measure the said X: now because $X \dots Y :: C \dots D$, that is O. P, also by Permutation $X \dots O :: Y \dots P$, therefore because O measures X, also P will measure Y; but also E will measure Y (seeing E, F are *g* the least in their Proportion; and that $E \dots F :: Y \dots Z$) therefore also S, the least which E and P measure, will measure Y, the greater the lesser, which is absurd. *g Hyp.*

By the same Artifice four Ratios, or more, in infinitum, may be continued in their least Terms;

PROP. V.

THe proportion, or ratio of plain numbers (AB. CD) the one to the other is composed of the ratios of their sides; that is of $A \dots C$, and $B \dots D$; or of $A \dots D$, and $B \dots C$.

For $B \times C = BC$, the Ratio of AB to CD is composed of the Ratios AB .. AC and BC .. CD, for $AB \dots AC :: BC \dots CD$; but the Ratio AB .. BC is $= a$ to the Ratio $A \dots C$ and the Ratio BC .. CD $= b$ Ratio $B \dots D$, therefore also the Ratio AB .. CD is composed of the Ratios of the sides $A \dots C$, and $B \dots D$ *a 17. 7. b ibid.*
W. W. Demonst.

PROP. VI.

IF Numbers how many soever are in continual proportion A, B, C, D, E, and the first A, doth not measure the second B, neither shall any of the other measure any one of the rest.

That none measures the next following is manifest from the hypothesis its self, but that any other doth not measure any one of the rest, I thus shew.

Let there be found *a* N, O, P, the least in the same proportion with A, B, C, D, E. *a 35. 7.*
Therefore N, P, are prime the one to the other and by *b* equality it will be *b 22. 7.*
 $A \dots C :: N \dots P$. but because $A \dots B :: N \dots O$, and that

E 2

A

A does not measure B, neither will N measure O, and consequently N is not a Unite.

A, B, C, D, E, Wherefore seeing N, P are prime the one to the other, N, will not

c def. 22.

c measure P, but $A \dots C :: N \dots P$.

wherefore A doth neither measure C, after the same manner I will shew that B doth neither measure D, the third number from its self, nor C, the third E from its self; and if four A, B, C, D, continual proportionals the least in their proportion be assumed, it may be demonstrated by the like way, that neither A doth measure the fourth D, nor B the fourth from it E, and so on.

PROP. VII.

IF there are Numbers, how many soever A, B, C, D, E, in continual proportion and if one of them measures any other; (not the second B,) also the first A, shall measure the second B.

a prac.

For if A does not measure B. neither shall any of the other *a* measure any of the rest, contrary to the hypothesis.

PROP. VIII.

IF four Numbers are in the same proportion ($A \dots B :: C \dots D$) as many mean proportionals as falls between the two first (A and B) so many shall also fall between the two last (C and D)

a 5, 7.

Between A, B, let the means P, Q. A, P, Q, B, fall, and find the *a* least proportionals N, O, R, S, N, O, R, S, to the numbers A, P, Q, B. C, V, X, D, therefore by *b* Equality it will be N.

b 22, 5.

c 3, 8.

d 22, 7.

e 21, 7.

$S :: A \dots B$ that is $C \dots D$; but N, S, are *c* prime the one to the other, and consequently the *d* least in their proportion, therefore N, S, equally measure *e* C, D proportional to them, as often as O and R, measure others V, X, wherefore because N, O, R, S, equally

equally measure the said C, V, X, D. it is manifest that C, V, X, D, are proportional to the said N, O, R, S, *f Hyp.* that is to *f* A, P, Q, B given, wherefore seeing A, P, Q, B are continual proportionals, also C, V, X, D are so many continual proportionals and consequently between A, and B, and C and D, there fall equal Number of mean proportionals, W. W. Demonstrated.

PROP. IX.

IF two Numbers C and D, are prime to one another, as many mean proportionals as fall between them, so many means fall between each of them and a Unite.

Between C and D let there be the mean proportionals O, P, find a A, B, the

two least in the Ratio of C to D, then the three AA, AB, BB, lastly the four

AAA, AAB, ABB, BBB, until the multitude of those found be equal to the multitude of the given C, O,

P, D wherefore because the extremes C, D, are *b* Prime one to the other, C, O, P, D are *c* the least in their proportion, that is, the least in the ratio *d* of A to B; wherefore seeing AAA, AAB, ABB, BBB are also *e* the least in the Ratio of A to B, these shall be equal to them, each to each; lastly from the 2 Prop. of this Book, and def. 14. it is manifest that Unity and A, AA, AAA, also Unity and B, BB, BBB, are continually proportional. Wherefore seeing that the number of terms AAA, AA, A, as well as BBB, BB, B, with Unity is equal to the number of terms AAA, AAB, ABB, BBB, that is C, O, P, D, as many means as fall between C and D, so many fall between a Unite and AAA or C; also between a Unite and BBB or D. which was to be demonstrated.

C, O, P, D

a 2, 8.

1.
Unity

A B
AA, AB, BB
AAA, AAB, ABB, BBB

b Hyp.

c 23, 7.

d Hyp.

e constr.

E 3

PROP.

PROP. X.

IF between a Unity and two Numbers (*AAA* and *BBB*) there fall an equal number of mean proportionals: there also fall the same number of means between the numbers *AAA*, and *BBB* themselves.

Let the whole construction of Prop; 2. of this Book be considered, and the Demonst. of this Prop. will be manifest from the 4 and 5 Corollaries of the said Prop.

Corollary.

Unity

1.

IF there are two ranks of continual proportionals, from Unity 1, *A*, *AA*, *AAA*, &c. 1. *B*, *BB*, *BBB*, &c. The Ratio of *AA* to *BB*, will be duplicate to that of *A* to *B*, and the Ratio of *AAA* to *BBB* will be triplicate of that of *A* to *B* and so on.

a Sch. p.

17, 7.

b def. 12.

c Sch. 17,

d def. 12.

For seeing that *AA*, *AB*, *BB*, are continual *a* proportionals in the reason of *A* to *B*, therefore the Ratio *AA* .. *BB* is duplicate of the *b* Ratio *AA* .. *AB*, that is of *A* .. *B*; likewise seeing *AAA*, *AAB*, *ABB*, *BBB* are continually proportional in the Ratio of *A* to *B*, the Ratio *AAA* .. *BBB* shall be triplicate of the Ratio *d* *AAA* .. *AAB*, that is of *A* .. *B*, and so on *W. W. D.*

2. The first Corollary will hold true, let the ranks or series begin from any common number *C*.

For seeing that *A*, *B*, *C* and *A*, *E*, *F* are in continual proportion, therefore *A* .. *B* = *E* .. *F*, but the Ratio of *BB* .. *EE* is duplicate of the Ratio *B* .. *E* by the 11 which does not depend on this, therefore also the Ratio

2c, 7:

b Sch. 17,

CA .. *FA* (that is the ratio *b* *C* to *F*) is duplicate of the

the Ratio B.. E, which was the first, in like manner I will shew, that the ratio DB.. GE is duplicate of the ratio C.. F, and consequently quadruplicate of the ratio B.. E; but the ratio DB.. GE, is composed of the ratios D.. G and B.. E, therefore the ratio D.. G with the ratio B.. E is quadruplicate of the ratio B.. E, therefore the sole ratio D.. G is triplicate of the ratio B.. E, which is the other.

3. If between the number A, and the two numbers D, G there fall equal number of mean proportionals B, C, E, F: also between the said D, G (although either of them be a Unire) there will fall the same number of means.

This is manifest from Corol. 2, for seeing the ratio D..G is triplicate of the ratio B..E, between D and G there falls two c mean proportionals in the ratio c def. 12. B..E.

PROP. XI.

Between two square numbers (AA, BB) there falls one mean proportional (AB) and the proportion of a square number to a square number is duplicate of the proportion of the sides (A, B)

Part. 1. AA..AB::a A..B and AA, AB, BB a Scho. 17. AB..BB::A..B therefore AA.. A. B. 7.

AB::AB..BB which was the first. o. 11. 5.

Part. 2. From the first Part, it is manifest that the b def. 12. ratio AA..BB is duplicate of the b ratio AA..AB; c Sch. 17. that is of the ratio c A..B. 7.

SCHOLIUM.

From this, and the 8 Prop. foregoing may be Demonstrated, that Celebrated Theor. which is the last of the Tenth of Euclid; in a square the diameter is incommensurable to the side.

For if it be denied, the diameter is to the side, as a number is to a number, Suppose as a to b , as is manifest from the def. of commensurable magnitudes, therefore also (as is manifest from 22. Eu. 6.) the square of the Diameter is to the square of the side, as a square number aa is to a square number bb , which cannot be, for seeing (as is manifest from 47. Eu. 1.) the square of the Diameter is to the square of the side as 2 is to 1. If that were to this as the square number aa is to a square number bb , also it would be $aa \dots bb :: 2 \dots 1$ and consequently seeing between aa and bb by this Prop. there falls one mean, a whole number, also between 2 and 1 there would fall one integral mean by the 8 Prop. of this Book which is absurd.

PROP. XII.

Between two Cube Numbers (AAA and BBB) there falls two mean proportionals (AAB , ABB) and the proportion of a Cube to a Cube, is triplicate of the proportion of their sides A , B .

AAA , AAB , ABB , BBB .

A , B ,

By the Scholium to Prop. 17. 7. AAA , AAB , ABB , BBB are continual proportionals in the reason of A to B , which was the first; from which and the 12 Def. the second is manifest.

PROP. XIII.

If there be given Numbers, how many soever, A , B , C , in continual proportion, which being multiplyed in themselves make AA , BB , CC , and being again multiplyed in these make, AAA , BBB , CCC and so on in infinitum.

I say that AA , BB , CC , and also AAA , BBB , CCC , and so on; are continually proportional. By Corol. 4. Prop. 2. of this Book, three ranks here beginning from Unity are continual proportionals. Therefore the

the ratio $AA \dots BB$ is duplicate of the ratio $A \dots B$, that is of the ratio $B \dots C$ but also the ratio $BB \dots CC$ is duplicate of the ratio $B \dots C$. Therefore $AA \dots BB :: BB \dots CC$; in like manner because the ratio $AAA \dots BBB$ is triplicate of the ratio $A \dots B$ that is of $B \dots C$, whose triplicate is also of the ratio of $BBB \dots CCC$, also it will be $AAA \dots BBB :: BBB \dots CCC$, which was to be demonstrated.

PROP. XIV.

IF a Square Number (AA) measure a square Number (BB), also the side (A) of one, shall measure the side (B) of the other.

And if the side A , measure the side B , also the square shall measure the square.

Part 1. $A \times B = AB$, by Schol. $AA \dots AB \dots BB$.

Prop. 19, Eu. 7. $AA \dots AB \dots BB$ are proportionals in the Ratio of A to B ; wherefore, seeing AA measures BB , also A shall measure AB ; But $AA \dots AB :: A \dots B$, therefore also A measures B , which was the first.

Part 2. $A \dots B :: A \dots AB$, but A is already put to measure B , therefore also AA measures AB , again, because $A \dots B :: A \dots AB$, and that A measure the said B , also AB shall measure BB , therefore also AA shall measure BB , which was the other.

PROP. XV.

IF a Cube number AAA , measures a Cube number BBB , also the side A shall measure the side B , and the contrary.

Part

Part 1 . Between both the Cubes, set the numbers

AAB, ABB, whose Genefis

AAA, AAB, ABB, BBB, the Letters sufficiently de-

Sch

A. 22 37 44 B. the letters sufficiently de-
clined: all four are in a con-

17. 7.

...proportion in the

8. 550

reason of A to B, wherefore seeing AAA measures B

C 7.8

BB, b also AAA shall measure cIAB, but AAA, A

d Scho

AB :: d A...B. therefore also, A measures B, which

17. 7.

was the first. A 1941 *Life* magazine photo of the

e ibid.

Part 2. Because $AA \vdash AAB$ and $A \vdash B$, and that

Fibid.

A is put to measure B, also AAA shall measure AAB:

in like manner, because AAB, ABB, BBB, f are one

unto another, as A is to B. also AAB shall measure A

BB, and ABB, the said BBB, therefore also : AAA

E. A. C.

measures BBB, which was the other.

PROP. XVL

IF the square (AA) doth not measure the square (BB) neither shall the side (A) measure the side (B,) and the contrary.

Part 1. For if the side A, doth

AA. BB. then measure the side B; also AA.

A. B. would measure BB. contrary to

2 2 pg

the Hypothesis

pro. 12

Part 2. If the side A not measuring B, AA mea-

6. 100

fures the said BB, it would also follow that b A would

the (an

measure B, contrary to the Hypothesis.

PROP. XVII.

IF the Cube AAA, does not measure the Cube DDD ;
neither shall the side A measure the side D, and the
contrary.

The is demonstrated from the absurdity of its contrary by Prop. 15,

PROP.

PROP. XVIII.

Between two like plain Numbers (AB, CD) there is one mean proportional Number, and the Ratio of a plain Number to a plain, is duplicate of the ratio of their Homologus sides (A, C .)

Part 1. Because AB, CD are like Plaines, it will be $a A..B::C..D$ AB, BC, CD , and by permutation $A..C::B..D$; $A. B. C. D$. seeing therefore that $AB..BC::$ $A..C$; that is (as I have already shewn) $B..D$; that is $d BC..CD$, BC shall be a mean Proportional between AB , & CD ; which was the first.

Part 2. By the first part, the Ratio of $A B$ to CD is Duplicate of the e Ratio of AB to BC ; that is of the Ratio of A to C , which was the other.

a Def. 27.
b 16 5
c Schol.
17 7.
d *ibid.*
e defin.
f Schol.
17 7.

PROP. XIX.

Between two like Solids (ABC, DEF) there falls two mean Proportionals (BCD, CDE) and the Ratio of a Solid to a Solid is Triplicate of the Ratio of their Homologus sides.

Part 1. Because the Solids ABC , and BEF , are similar, or like the sides; $A B, C$ of one are Proportional to the sides D, E, F , $A. B. C.$ of the other, wherefore by Permutation $A..D::B..E$ and $C..F$. now by the Schol. to Prop. 17. Eu. 7. $ABC..BCD::A..D$; $BCD..CDE::B..E$; $CDE..DEF::C..F$. but I have already shewn that $A..D::B..E::C..F$. therefore also $ABC..BCD::BCD..CDE::CDE..DEF$, and consequently BCD, CDE are two mean Proportionals, between ABC , and DEF , which was to be Demonstrated.

Part

Part 2. This is already manifest from the first part, and the 12 Def. viz. That the Reason of ABC to DEF is triplicate of the reason of ABC to BCD; that is 2 of the Reason of A to D, which was to be demonstrated.

Scholi.
17 7.

Scholium.

THis place requires that we demonstrate, that when a Solid Number is produced of three Numbers, or of Numbers how many soever, multiplied any how the one by the other, they do always produce the same Number.

This is an excellent Theorem, and of great use, which we will demonstrate otherways, and far more expedite then has been done by others; and because it depends on the permutations which can be made of a given Number of things; we will premise this.

Theorem 1.

LET there be given Things or Letters, A, B, C, D, E, as many as you please; and take so many Numbers from Unity 1, 2, 3, 4, 5, as there are Letters, which multiply one into the other orderly, and the Product shall be the Number of Permutations, which can be made of the given things, A, B, C, D, E.

Two Letters (which we assume for things) a, b, may be twice changed, each standing the first or last place once; hence,

Three Letters a, b, c, may have six permutations, for any one of the three occupying the third place, the other two may be changed twice, for when c is in the last place, the other two a b, may be changed twice; and so there will be had the two divers orders, abc, bac. Again if b is in the last place, the others a c, may be twice changed, and so there will arise the two new Orders, acb, cab: lastly, if a be in the last place, the remaining b c, have two per-

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45

permutations, whence again arise two other orders, bca, cba: see them together.

abc acb bca
bac cab cba

Four Letters, a, b, c, d, have 24 permutations, for any of the four Letters may once be in the last place, and then the other three may be permuted 6 times, whence there will arise 24 several Orders of the four Letters.

Five Letters, a, b, c, d, e, may have 120 permutations; for any of the five being in the last place, the other four may be permuted 24 times, whence there arise 5 times 24 several orders of the 5 Letters given, which is 120, and so in infinitum. This being promised, we proceed to

Theorem 2.

Part, 1. **T**hree Numbers A, B, C, being in any order, multiplied the one by the other, always produce equal Numbers.

The Table here set shews six several Orders of Multiplications. That the Products are equal when the same Letter is in the least place, as in aBc and bac, &c. is manifest from 16 E. 7. Therefore it remains to shew that aBc, aCb, bCa, are equal.

Let us compare the first aBc and aCb; aB..a aC::B..C but B and C are the same numbers as b and c, therefore aB..a aC::b..c; therefore b a B x c (= a B c) = a C x b (= a C b); Now let us compare aBc and bCa, aB

aBc
bae

aCb
cab

bCa
cba

aBc
aCb

aBc
bCa

..bC

$1. b C : 1 a . C ;$ that is $A . C ;$ therefore the Product $a B c = b C a$, therefore it is manifest that all the six Products are equal to one another.

Part 2. *Likewise 4 Numbers, a, b, c, d, being multiplied in any order, the one into the other, produce equal Numbers.*

By Theor. 1. It is manifest that a, b, c, d, hath 4, Senaries of different Orders, that is 24 different Orders; the first Senary is exhibited in the apposed Table; because a, b, c, multiplied the one into the other any way, always produce the same Number by the first part hereof; and the last in the whole Senary is the same; it is manifest that all the Products of the first Senary are one and the same, or equal: After the same manner may be shewn that the Products of the second Senary agree; and the like may be demonstrated of the third and fourth Senary. Now this only remains, that the Product of one Senary agreeth with any other of the three remaining Senaries, which I thus shew;

Let us compare the first Senary in which d is in the first place, with the fourth Senary, in which a is in the last place; write a under d, then before d put A, and D before a, and set the other Letters bc, before both; therefore $bcA . . bcD : : A . . D$; that is $a . . d$; therefore the Products $bcAd$, and $bcDa$ of the first and fourth Senary are equal. In like manner I will shew that the Products of the 4 Senarys agree, therefore the proposed is manifest.

Part 3. *By the same Ratiocination by Part 2. may be shewn, that the Products arising from the Multiplication of 4 Numbers, which are 120, are the same, and so on in infinitum in 6, 7, 8 &c. Numbers.*

PROP.

PROP. XX.

IF between two plain Numbers (*A* and *B*) there falls one mean Proportional *C*, they are like plain Numbers.

Assume *F, K* the least in the ratio of *A..C* and *C..B*; then *F, K* equally measure as well

the said *A, C*. suppose by *X*; as the

said *C, B*, suppose by *Z*, therefore *c K*

$x X = C$ and $c K x Z = B$, and conse-

quently $X..Z::d C..B::F..K$, and by

permutation $F..X::E..Z$. wherefore seeing $e F$

$x X = A$, and $K x Z = B$, and that consequently *F, X*, and *K, Z*, are the sides of the Planes *A* and *B*,

therefore *A, B* are like *f* plain Numbers, which was

to be Demonstrated.

a 35 7
b 21 7
c Ax. 8
d 17 7
e Hyp. 8
f Def. 27

Something otherways.

IF between two Numbers, (*A, B*) there falls one mean Proportional *C*, they are like plain Numbers.

Take *a D, E* the least in the ratio *A..C* and *C..B*.

B, therefore *D, E* equally mea-

sure as well *A, C*, suppose by *M*,

as *C B* suppose by *N*. wherefore *D*

$x M = DM = c A$, and $E M d =$

C, and $e E N = B$; now in *DM*

and *EM*; $D..E::f DM..EM$:

$: A..C::g C..B::E M..E N::h M..N$;

therefore *D M, E N*, that is *A, B*, are *i* like plaine

Numbers which was to be Demonstrated.

a 35 7
b 21 7
c Ax. 8
d the same
e the same
f Sco. 17
g Hyp.
h Schol. 17
i Def. 27

PROP.

PROP. XXI

IF between two Numbers (A, B) there fall two mean Proportionals (C, D) the Numbers (A, B) are like Solid Numbers.

a 35 7 Let there be taken E P, X, O Q, the *a* three least proportionals to A, C, D

b Cor. 21 which may also equally *b* A. C. D. B
7 measure the said A, C, D. EP X OQ

suppose by M; and consequently E P M (= E P x M, XM, XN OQN.
c Ax. 8 M) = *c* A and XM = *d*

d the same C. but because C, D, B, are in the same reason, as A, C, D; thence EP, X, OQ are the least Proportionals to the said C, D, B, and consequently equally

e Cor. 21. 7 *e* measure them, suppose by N, wherefore XN (= *f* Ax. 8 X x N) = *f* D and OQN = B. Seeing therefore, that EPM = A and OQN = B, it remaineth to shew, that the sides E, P, M are Proportional to the sides O, Q, N.

g Constr. Because between EP and OQ there is a *g* mean Proportional X. the said EP, OQ *h* are like Plains,

i Constr. therefore *i* E..O::P..Q; then M..N::*k* XM..
k Sc. 17. 7 XN; (that is as I have already shewn above) C..D

l Constr. that is *l* EP..X, but EP..X::P..Q; because both *m* EP..X and *n* P..Q are in half the Ratio of

m Defi *n* 18 8 EP..OQ, therefore *o* P..Q (that is E..O)::M
o 11 12 ..N: And therefore the Solids EPM, OQN, that

p Def. 27. is A, B are *p* like Solid Numbers- W. W. to be demonstrated.

PROP.

PROP. XXII.

IF three Numbers (A, B, C) are Proportional, and the first (A) be a Square, the third also shall be a Square.

Let the root or side of the Square A be O ; because therefore, $O \times O = A$, it will follow, $1 \dots O :: a \ O :: A$; and consequently both between A and 1 , as also between A and O , there falls one mean Proportional; therefore also b between 1 and C falls a mean Proportional P , which being multiplied into its self c makes C , therefore also C is a Square Number. *W. W. Demonstrated.*

$A: B. C. a$ Def. 13

$O \ P$

I

b Cor. 3.

$10. 8.$

c Def. 13

d Def. 28

Corollary.

A Square, the Root, and Unity, are continual Proportionals; which appears from the Demonstration foregoing.

PROP. XXIII.

IF of four Numbers (A, B, C, D) continually Proportional, the first (A) be a Cube, also the fourth (D) shall be a Cube.

Let the side of the Cube A , be O , then $O \times O \times O = A$, and $O \times P = a \ A$, the Cube; now by Cor. foregoing $1 \dots O :: O \dots P$, and because $O \times P = A$, it will be $1 \dots O :: b \ P \dots A$; therefore both between A and 1 , and between A and D , there fall two mean Proportionals; therefore also between 1 and D , there falls two means Q, R ; seeing therefore $1 \dots Q :: R \dots D$, it

A, B, C, D a Def. 29

$P \ R$

$O \ Q$

b Def. 13

c Cor. 3.

$10. 8.$

d Def. 13

e Def. 13 it is manifest that $e Q \times R = D$. Therefore also f D
 f Def. 29 is a Cube Number.

Corollary.

UNity, the Root, the Square, and the Cube, are
 Continual Proportionals, which is manifest from
 what is Demonstrated.

PROP. XXIV.

IF two Numbers (A, B) have the same Proportion
 one to the other, as a Square Number C , hath
 to a Square D , and if the first A be a
 square, the second B , shall also be a square Num-
 ber.

Because $A \dots B :: C \dots D$, and be-
 tween C and D there falls one mean
 $C : P : D$ a Proportional P , also between A
 and B , there falls one mean O , because
 therefore the first A is a Square, the second also B
 shall be a Square. W. W. Demonstrated.

PROP. XXV.

IF two Numbers (A, B) have the same reason the
 one to the other, as a Cube C hath to a Cube D ; and
 if the first A be a Cube, the second B also shall be a Cube
 Number.

Because $A \dots B :: C \dots D$, and that between the
 Cubes C, D , there fall two
 mean Proportionals Q, R ; also
 between A, B , will fall two
 means; now because A is a
 Cube,

BOOK II. of Arithmetick.

57

Cube, therefore also B will be c a Cube. W. W. c 23 8
Demonstrated.

Corollaries.

1. **T**HE Ratio of a Square Number to a Number that is not a Square, cannot be express in two squares; as is evident from Prop. 24.
2. The Ratio of a Cube Number to a Number that is not a Cube, cannot be express in two Cubes; as is manifest from Prop. 25.

PROP. XXVI.

LIKE plain Numbers (A, B) have the same Proportion the one to the other, as a square Number hath to a square Number.

Between A and B there falls one a mean Proportional, which let be C, find the three least Terms D, E, F, in the Ratio's of A..C A. C. B. or C..B the Extrems D, F, are D. E. F. b Cor. 1
E Squares; because A..C :: c D.. 2 8
E and C..B :: d E..F: Therefore by equality c Constr.
lity e A..B :: □ D..□ F. which was to be demonstrated. d Constr. e 22 7

F 2

PROP.

PROP. XXVII.

LIKE Solid Numbers, (A, B) have the same Reason the one to the other, as a Cube to a Cube Number.

19 8

Between A and B there falls two mean Proportionals, which let be C, D;

A. C. D. B

E. G. H. F

Cor. I 2

find the four least Terms, E, G, H, F, in the Ratio of A..C; the Extremes E and F are Cubes; and by equality it will be A..B:: Cube E. Cube F. W. W. Demonstrated.

PROP. XXVI.

~~THE~~

THE

THE
THIRD BOOK
OF THE
ELEMENTS
OF

Arithmetick,
And the Ninth of
EUCLID.

PROP. I.

TWO like Plains (*A, B,*) being multiplied the one into the other, produce a Square Number (*AB.*)

$A \times A = AA$, therefore it will be $AA :: AB ::$
 $A :: B$, but between b *A* and *B*, there falls one mean *b*

8 8 8

A: O. B.
AA. P. AB

Proportional, seeing A and B are like Plains: Therefore also between c AA and AB will fall the mean P; wherefore seeing the first AA is a Square by Construction, also the third AB shall be a Square; which was to be Demonstrated.

PROP. II.

11 8

Sch, 17.7

8 8

20. 8

IF two Numbers (A, B) being multiplied together, produce a Square (AB) those two Numbers are like plain Numbers.

$A \times A = AA$, because AA, and AB are both Squares, there falleth a a mean Proportion between them, but AA ..

A. O. B.
AA. P. AB,

BB: : A .. B; therefore also between A and B, there falls a c mean

Proportional. Therefore A, B, are d like plain Numbers. W. W. D.

Corollaries

18 8

22 8

1. **T**WO Squares produce a Square, for they are like Plains; therefore by the first hereof.

2. If two Numbers A, B, produce a Square, and one of them (suppose A) be a Square, the other also B shall be a Square.

For by the Second of this Book, A and B are like Plains; therefore between a A, B there falls the mean Proportional; seeing therefore the first A is a Square, also the third B b shall be a Square.

3. If two Numbers (A and B) produce a Number that is not a Square, although A be a Square, yet B shall not be a Square.

For if B were also a Square, then the two A and B, would produce a Square Number by Corol. 1. contrary to the Hypothesis.

4. The Square A, and B which is not a Square, produce a Number that is not a Square, otherway by Cor.

BOOK III. of Arithmetick.

35

Cor. 2. B would also be a Square, contrary to the Hypothesis.

PROP. XII.

A Cube Number (CCC) being multiplied into its self, produces a Cube D.

Let the Root or Side of the Cube be C, then by Cor. to Prop. 23. Eu. 8. 1, C, CC, CCC, are in continual Proportion; and therefore between 1, and CCC, there falls two mean Proportionals, but because $CCC \times CCC = D$, it will follow $1 : CCC :: CCC : D$; therefore also between b CCC and D there falls two means, wherefore seeing the first CCC is a Cube, also the fourth D will be a Cube, which was to be Demonstrated.

CCC

CC

C

D

a Def. 13

b Prop. 89

c 23 8

PROP. IV.

IF a Cube A be multiplied by a Cube B, the product AB, shall be a Cube.

$A \times A = AA$ and AA is a Cube; and because A and B are Cubes, there falls two mean Proportionals between them; but $AA : AB :: A : B$, therefore also between AA and AB there falls two means; wherefore because the first A is a Cube, the fourth AB. also shall be a Cube, demonstrated.

a Prac.

b 12 8

A. B.

AA AB

c Sch. 17 7

d 8 8

e 23 8

W. W. De-

PROP. V.

IF a Cube A, multiplying any other Number B, generates a Cube AB; also the Number B multiplied shall be a Cube.

Cube $A \times A = AA$, but AA will be a Cube; a 3 9

F 4

now

§ 12 8

A. B
AA. BB

c Sch. 17 7

d 8 8

e 8 8

now because AA and AB are both Cubes, there will fall two *b* mean Proportionals between them, but AA . . AB :: c A . . B; therefore also between A *d* and B there falls two means; wherefore because the first A *e* is a Cube, the fourth B shall also be a Cube. W. W. Demonstrated.

Corollary.

1. **O**F a Cube (A) multiplied by a Number X that is not a Cube, is made a Number that is not a Cube; for otherwise by Prop. 5. also X would be a Cube, contrary to the Hypothesis.

2. If a Cube A multiplied by B maketh a Number that is not a Cube, neither shall B be a Cube; otherwise by Prop. 4. also the Fact of A x B would be a Cube, contrary to the Hypothesis.

PROP. VI.

IF a Number A multiplying its self, produces a Cube B, that Number A its self is a Cube.

Def. 29

A x B = E, because A x A = B, and A x B = E, it is manifest that E is a Cube; and because that A x B Cube, = E Cube, also the Cube B x A = b E Cube, therefore also c A is a Cube, which was to be Demonstrated.

PROP. VII.

IF a Composi Number A multiply any other Number B, the Number produced E shall be a Solid Number.

Some

Some Number (besides a Unit) measure a Com-
 posit Number (which let be C) by some
 other Number, which let be D; *a* there- *a* Def. 22
 fore $C \times D = b$ A but $A \times B = c$ E; C, D *b* Ax. 8
 therefore E is made from the Multiplica- *c* Hyp.
 tion of the three Numbers C, D, B. therefore E is *d* Def. 25
 a *d* Solid Number, which was to be Demonstra-
 ted.

Lemma.

IN a Series of Numbers, continually Proportional and
 beginning at Unity (1, A, B, C, D, E, &c.) The
 first Number A from Unity, multiplying any of the other
 D, produces the next following Number E.

For it will be f $1..A::D..E$; from whence f Def. 13
 this is manifest.

PROP. VIII,

IF from a Unite there be Numbers how many soever,
 continually Proportional, the second Number B (Unity
 not being reckoned) shall be a Square, and so all follow-
 ing D, F, N, &c. leaving one between.

But the third C is a Cube, and so all the follow-
 ing F, K, &c. intermitting two.

The sixth F, is both a Cube Number, and a
 Square and so all the following, intermitting
 five;

1 2 3 4 5 6 7 8 9 10 11 12 13

1. A, B, C, D, E, F, G, H, K, L, M, N, O.

1. 2. 4. 8. 16. 32. 64. 128.

Part. 1. Because $1..A::A..B$ it is manifest that *a* Def. 13
 $A \times A = a$ B, and consequently *a* B is a Square. *a* Def. 28
 Also because B, C, D are continual Proportionals;
 and that B is a Square, also *b* D shall be a Square, *b* 22 8
 and so on intermitting one.

Part 2.

c Lem.

d Def. 29

e 23 8

Part. 2. Because $A \times A = B$, and again $A \times B = C$, therefore d C shall be a Cube: Lastly because C, D, E, F, are four continual Proportionals, and that C is a Cube, also F shall be a Cube; and so on, intermitting always two.

Part. 3. This is manifest from Part 1 and 2.

SCHOLIUM.

Because the first Number in a Series of continual Proportionals, multiplying any other in the said Series, produces the next following; it is manifest that a Series of continual Proportionals, may excellently be express'd, only by the continual setting together of the Letter A, representing or designing any Number, after this manner.

1	2	3	4	5	6
I. A.	AA.	AAA.	AAAA.	AAAAA.	AAAAAA.
A. 7.	A. 8.	A. 9.	A. 10.	A. 11.	A. 12.

That is $A \times A = AA$. a Square; $A \times AA = AAA$ a Cube; and so on, whence the whole Demonstration of the Propositions is seen by once looking on; for because the Product of Multiplication is shewn by the putting together of Letters, it is manifest that the Product will be a Square, when the number of Letters is even, which consequently a Binary will measure, as is done in the second, third, and fourth place, and in all the following, intermitting one.

Again it is manifest that the Product is a Cube, when Three or a Ternary measures the Number of Letters; and so may be Trisected, which happens in the third and sixth place, and in all the following, intermitting always two: Lastly it is manifest, that when the multitude of Letters is a prime Number, as 5, 7, 11, 13, &c. that it hath neither a Cube nor a Square; for seeing no Number measures prime Numbers, the Number of Letters cannot be Bisected,

ed, as is required to a Quadratick, nor Trisected as to a Cubical multiplication: These Numbers are usually by Arithmeticians called Surdesolids or Super-solids, but all the other terms of the Progression are Squares or Cubes, or both; because that seeing the Numbers of Letters or Dimensions, whereof they consist, are composited Numbers, either a Binary or Ternary, or both together, measure them, and consequently they may be either Bisected or Trisected, or both.

Lastly, The Number of Letters in the Terms of the Progression already spoken of are called Porestates, and are wrote over them in a Natural order, which shew the place or Dimensions of each, and are also called Exponents; so in the foregoing Series, which we used before the Scholium: A is the first Term, and is called the Root; B is the Second or of two Dimensions, and is called a Square, C the third or of three Dimensions, is called a Cube, &c. In the other Series the Number of Letters, express the Place, the Sides, and Dimensions of each power.

From what is above said it is manifest, 1. That in every Series of continual Proportionals, the Term whose exponent is 6, is a Square and a Cube together, for the Number of its Sides, seeing it can be measured by 2 and 3, may be Bisected and Trisected. 2. That intermitting always 5, all the rest are together Cubes and Squares, to wit the 12, 18, 24. Term, and the rest whose Exponent's a Senary measures; for seeing both 2 and 3 measures 6, and that 6 measures 12, 18, 24, &c. also as well 2 as 3 shall measure the same, which consequently may be Bisected and Trisected; therefore, &c.

PROP. IX.

IF in a Series of Numbers continually Proportional from a Unite, the first Number A be a Square, all the rest shall be also Squares.

Q. E. D.

c Lem.

d Def. 29

e 23 8

Part. 2. Because $A \times A = B$, and again $A \times B = C$, therefore d C shall be a Cube: Lastly because C, D, E, F, are four continual Proportionals, and that C is a Cube, also F shall be a Cube; and so on, intermitting always two.

Part. 3. This is manifest from Part 1 and 2.

SCHOLIUM.

Because the first Number in a Series of continual Proportionals, multiplying any other in the said Series, produces the next following; it is manifest that a Series of continual Proportionals, may excellently be express'd, only by the continual setting together of the Letter A, representing or designing any Number, after this manner,

●	1	2	3	4	5	6
I.	A.	AA.	AAA.	AAAA.	AAAAA.	AAAAAA.
A.	7.	A. 8.	A. 9.	A. 10.	A. 10.	

That is $A \times A = AA$. a Square; $A \times AA = AAA$ a Cube; and so on, whence the whole Demonstration of the Propositions is seen by once looking on; for because the Product of Multiplication is shewn by the putting together of Letters, it is manifest that the Product will be a Square, when the number of Letters is even, which consequently a Binary will measure, as is done in the second, third, and fourth place, and in all the following, intermitting one.

Again it is manifest that the Product is a Cube, when Three or a Ternary measures the Number of Letters, and so may be Trisected, which happens in the third and sixth place, and in all the following, intermitting always two: Lastly it is manifest, that when the multitude of Letters is a prime Number, as 5, 7, 11, 13, &c. that it hath neither a Cube nor a Square; for seeing no Number measures prime Numbers, the Number of Letters cannot be Bisected,

ed, as is required to a Quadratick, nor Trisected as to a Cubical multiplication: These Numbers are usually by Arithmeticians called Surdefolids or Super-solids, but all the other terms of the Progression are Squares or Cubes, or both; because that seeing the Numbers of Letters or Dimensions, whereof they consist, are composited Numbers, either a Binary or Ternary, or both together, measure them, and consequently they may be either Bisected or Trisected, or both.

Lastly, The Number of Letters in the Terms of the Progression already spoken of are called Potestates, and are wrote over them in a Natural order, which shew the place or Dimensions of each, and are also called Exponents; so in the foregoing Series, which we used before the Scholium: A is the first Term, and is called the Root; B is the Second or of two Dimensions, and is called a Square, C the third or of three Dimensions, is called a Cube, &c. In the other Series the Number of Letters, express the Place, the Sides, and Dimensions of each power.

From what is above said it is manifest, 1. That in every Series of continual Proportionals, the Term whose exponent is 6, is a Square and a Cube together, for the Number of its Sides, seeing it can be measured by 2 and 3, may be Bisected and Trisected. 2. That intermitting always 5, all the rest are together Cubes and Squares, to wit the 12, 18, 24. Term, and the rest whose Exponent's a Senary measures; for seeing both 2 and 3 measures 6, and that 6 measures 12, 18, 24, &c. also as well 2 as 3 shall measure the same, which consequently may be Bisected and Trisected; therefore, &c.

PROP. IX.

I*n a Series of Numbers continually Proportional from a Unite, the first Number A be a Square, all the rest shall be also Squares.*

If the first is a Cube, all the rest shall also be Cubes.

Part 1. Because A by Hypothesis, and B by the preceeding are Squares, and that $A \times B = C$ also a C shall be a Square; again because $\square A \times \square C = b D$ also D shall be a Square, and so on.

a Cor. 1, 2.

9.

b Lemn.

c 3. 9.

d Lemn.

e 4. 9.

Part 2. Because A by Hypothesis is a Cube, and that $A \times A = B$; also B shall be a Cube, and because the Cube $A \times \text{Cube } B = d C$, also C shall be a Cube, &c.

PROP. X.

IF of Numbers continually Proportional from a Unite, the first A be not a Square; neither shall any other be a Square, but the second B, and the rest following D, F. intermitting always one Term.

And if the first is not a Cube; neither shall any other be a Cube, but the third, and the rest following R &c. intermitting two always.

Part 1. If it can be, let E

1. A. B. C. D. E. F. G. be a Square Number; therefore because $E \dots D :: B \dots$

a Hyp.

b 18. 9.

c 24. 8.

d 8. 9.

e 25. 8;

A, and that $a E b D b B$ are Squares, also A shall be a Square, contrary to the Hypothesis.

Part 2. If it be possible let D. be a Cube, seeing therefore d F and C are also Cubes, and that by Hypothesis and equality of Reason it is $F :: C :: D$. A, also e A shall be a Cube, contrary to the Hypothesis.

PROP. XI.

IN a Series of Numbers continually Proportional from Unity, any of the lesser Terms C, measureth the greater G. by some Numbers which is in the Series.

1. A. B; C. D. E. F. G. for by equality of Reason, a Def. 19, $C \dots G :: I \dots D$, therefore C measureth a G by D.

Corol.

Corollaries.

1. **H**OW far the greater Number G is distant from the Number C which measures it, so far is D the Number by which the lesser Measures the greater distant from Unity.

2. The first Number A measures any other Number D. by that which goes before it C.

3. Any Number C in a Series, multiplying its self, produces a Number F so far distant from its self, as its self is distant from Unity.

4. If any Number B, in a Series multiplyeth any other Number D in that Series, as far as Unity is distant from the lesser B, so far the greater D shall be distant from the Product F.

PROP. XII.

IF from a Unite Numbers how many soever are continually Proportional, A. B. C. D. the prime Number (E) that measures the last D, shall also measure A that is next to the Unite.

For, if it be possible, let the prime E, not measure A; therefore A and E are Primes, the one to the other, but because E measures D, let it be by F, and A measures the same D by C, therefore $E : A :: C : F$; but because E and A are Prime the one to the other, they are the least in their Proportion; therefore E measures C, suppose by G. Wherefore seeing A measureth the same C by B, it will again be $E : A :: G : B$; G therefore because E, A, are the least in their Proportion: E again measures B suppose by H; but also A measures B by I Cor. 19. A (for $A \times A = D$, as is manifest from the 8) therefore again $E : A :: A : H$; wherefore seeing E, A are the least in their Proportion E measureth A, which was to be Demonstrated.

PROP. k 25. 7.

PROP. XII.

If from a Unite be Numbers continually Proportional, and the next to the Unite A be a Prime Number, then no other Number shall measure the greatest D, but those which are among the Proportional Numbers.

1. For, if it be possible, let another E different from the said A, B, C, I, A, B, C, D, measure the greater D, but E must not be a Prime; otherwise E would also measure A; and consequently A would not be a Prime contrary to the Hypothesis; therefore E is composi, and so some Number measures it.

2. Which I say is A, for if any other Prime O, measure E, then because E measures D; also O would measure D, therefore also the said A, which is absurd, seeing A is a Prime. Now let E measure D by H.

3. H, shall be different from A, B, C, for if it be possible let H be the same with any of the said A, B, C, suppose C, therefore E measures D by C, and consequently C also measures D by E, therefore E is one of the Rank A, B, C, which is absurd; seeing you would have E not to be one of the Rank A, B, C, therefore H is different from the said A, B, C.

4. Also H will not be a Prime Number; otherwise seeing H measureth D, it would also measure A, which is Prime by the Hypothesis, which is absurd; Therefore H is a composi Number.

5. But because H is a composi Number, some Prime Number will measure it, which I say is A, for if any other Prime O should measure H, seeing H measureth D (for because E measures D by H, also H shall measure D by E), also O shall measure the said

said D, and consequently also the said k A a Prime k *Prac.*
which is absurd.

6. Now because A measureth l D by C, and that
also E measureth D by H, it will be $A \dots E :: m H \dots C$,
wherefore seeing (as I shew in Number 1) A measures
E, also H shall measure C, suppose by G; because there-
fore, as E, being divers from A, B, C, did measure
the last D by H; so now H which I have already
shewn in Numb. 3. to be diverse from A, B, C, mea-
sures the last C of the said A, B, C by G; after the
same manner I will shew that G is distinct from A, B,
and that it is not a Prime; and that it may be measur-
ed by A; which three I have shewn of the Numb. H.

Therefore because A measureth C, n by B, and H
measureth C by G, it will be $A \dots H :: G \dots B$. but I
have shewn Number 5. that A measures H, therefore
also G measures B, suppose by F. I may again shew
that F is different from A, as I have plainly shewn in
Numb. 3. that H is different from A, B, C, and G.
from A, B. Wherefore because A measures B by A,
and G measures B by F, it will be $A \dots G :: p F \dots A$; 7.
but it is shewn Numb. 6. that A measures G; there-
fore also F measures A, which is absurd, seeing A is a
Prime Number. But this absurdity is deduced from
this, that E is put different from A, B, C, and to mea-
sure D; therefore it is manifest that no Number dis-
tinct from A, B, C measures D, which was to be De-
monstrated.

PROP. XIV.

THE least Number A which certain Prime Num-
bers given B, C, D do measure, can be measur-
ed by no others than those given.

a 21. 8. *b* 32. 7. *c* 32. 7. *d* 32. 7. *e* 30. 7. *f* 26. 7. *g* 4. 2. *h* 27. 7. *i* 30. 7. *k* *ibid.*

A. If it be possible let some other Prime
B. C. D E measure the least A by. F therefore E
E. F $\times F = A$ and consequently E, F. are
the Sides of the said A, because there-
fore B, C, D measure A, they shall also measure
one of the said E, F, not E, the Prime; therefore F;
but F is less then A; for $F \times E = A$; therefore A is
not the least which B, C, D measure, which is against
the Hypothesis.

PROP. XV.

IF three Numbers AA, AB, BB, continually Proportional, be the least in their Proportion, any two added together shall be Prime to that which is left.

From Prop. 2, 8. it is manifest that,

A. B. if A, B are supposed to be the least
AA. AB. BB. in the Ratio given, that then AA. AB.
BB. shall be the three least in the same

Proportion. Now because A and B are a Prime the
one to the other; also $A \div B$ is Prime to B, but
also A is Prime to B, therefore also $A \div B \times A (=$
 $AA \div AB)$ is Prime to B; wherefore $AA \div AB$ shall
be also a Prime to BB; after the same manner
may be shewn that $BB \div AB$ is Prime to AA.

It remaineth to shew that $AA \div BB$ is Prime to
AB, because A, B are Prime the one to the other;
and so also as well A as B is Prime to $A \div B$; also
 $AB = A \times B$ shall be f Prime to $A \div B$; therefore
AB is g Prime to the Square of the said $A \div B$ that is
to $h AA \div BB \div AB$, therefore by dividing, AB is
Prime to $i AA \div BB \div AB$; and by k dividing again,
AB is Prime to $AA \div BB$. W. W. Demonstrated.

PROP. XVI.

TO two Numbers A and B that are Prime the one
to the other, there cannot be given a third Num-
ber in continual Proportion with the π .

For if it be possible, let it be $A \dots B$.
 $B \dots b \dots C$ because A and B are Prime the one to the other, they are the a least in their Proportion; therefore A measures b ; but also A measures its self; therefore A and b are composed the one to the other, contrary to the Hypothesis.

PROP. XVII.

IF in a Series of Numbers continually Proportional, the Extremes A, D are Prime the one to the other; the said Series cannot be continued further.

For if it be possible let it be $A \dots B \dots C \dots D$ so D to some other E ; therefore by Permutation $A \dots D \dots B \dots E$, but A and D are Prime one to the other, and consequently the b least in their Proportion; therefore A measures B , but B measures C , and C measures D ; therefore A measures D , but A also measures it self; therefore A and D are composed the one to the other, which is against the Hypothesis.

PROP. XVIII.

TWO Numbers A, B being given to consider, whether there can be found a third Proportional to them.

If A, B are Prime the one to the other, there cannot be found a third which is manifest from the 16. Prop.

If A, B are not Prime the one to the other; $B \times B = BB$, if A measures BB , suppose by X , then X shall be a third Proportional, for $X \times A = BB$; and also $b \times B = BB$; therefore $b A \dots B \dots X$.

a the same
d Ax. 7. A. B. Z. If A doth not measure BB, a third
 BB. Proportional cannot be had.
 If it possible, let Z be it; there-
 fore $A \times Z = BB = c B \times B$ therefore because
 $A \times Z = BB$, A shall measure $d BB$ by Z, contrary
 to the Hypothesis.

PROP. XIX.

THREE Numbers being given A, B, C, to consider
 whether there can be found a fourth Proportional to
 them.

The second B x third C = BC, if
 A. B. C. X the first A measures BC, suppose by
 BC. X; then X will be a fourth Proportional,
 for then $A \times X = BC$ as well
a Ax. 8. as $B \times C = BC$; therefore $b A \dots B :: C \dots X$.
b 19. 7. If A does not measure BC, a fourth
 A. B. C. Z Proportional cannot be gotten; if it
 BC. can let Z be the Number, then $A \times Z$
c 19. 7. $= c BC = B \times C$; therefore A *d* mea-
d Ax. 7. sures BC by Z, contrary to the Hypothesis.

SCHOLIUM.

THE 4 preceding Propositions are to be understood of
 whole Numbers; for to any two Numbers, a
 third, and to any three a fourth Proportional may be found
 in Fractions, as shall be taught in Practical Arith-
 metick.

But this is worthy Observation, which we will De-
 monstrate in the 7 Chap. Book 5. of Practical Arithme-
 tick; that although to any two a third Proportional, and
 to three a fourth may be found, at least in a Fraction, yet be-
 tween those that an Integral mean cannot be found, nei-
 ther can a Fractional mean be found.

Three Numbers A, B, C, being given a
 A. B. C. Z fourth Proportional is also found in this
 D. manner.

The

The first *A* dividing the second *B* maketh the Quotient *D*, the third *C* multiplying *D* generates *Z*, which is that sought.

For seeing *A* divides *B* by *D*, also $a \times D = B$, but $a \times A \times 8$ also $C \times D = b \times Z$, therefore $A \dots B :: C \dots Z$. b 17. 7.

It will be impossible to find a fourth Proportional $c \times 7. 7$ which is an Integer (not if *A* does not measure *B* but) if the fact of *C* and *D* be not an Integer, or reducible to an Integer.

PROP. XX.

PPrime Numbers are infinite or there are infinite Number of Prime Numbers a. a 38. 7.

Let there be given three Prime Numbers *A. B. C.* find the least *X*, which *A. B. C* *A, B, C*, measure; to which add one, $X + 1$ if then $X + 1$ makes a Prime Number, *Z* you will have a fourth Prime: If $X + 1$ does not make a Prime, some *b* Prime Number will $b \times 34. 7.$ measure it, suppose *Z*; this will be a Prime different from the given *A, B, C*, for if it be possible let it be the same with one of the given Primes *C*; therefore because *C* measures *X*, also *Z* will measure *X*, but also *Z* measures $X + 1$. Therefore *Z* will measure $c \times 1$, also which is absurd; therefore *Z* is a new Prime; after the same manner may be found more, $c \times 12.$ infinitely.

PROP. XXI.

Even Numbers how many soever *A, B, C*, being added together, make an even Number.

Because *A, B, C* are even Numbers, let their *a* halves be *N, O, P*. Therefore because $A \dots N :: B \dots O :: C \dots P$ A. B. C a D: f. 16 $\dots P$ will also be $A \dots N :: b \times A + B$ b 12. 9 $+ C \dots N + O + P$, but *A* is the double of *N*; therefore $A + B + C$ is also the double of $N + O + P$. and

consequently $N + Q + P$ is half of $A + B + C$; therefore $A + B + C$ is an even Number, which was to be Demonstrated.

PROP. XXII.

Odd Numbers A, B, C, D , even in Number, compose an even Number.

Take a Unite from each, then what remains shall be a even Numbers, and the Unites taken away will compose or make an b even Number; wherefore by the preceding the Numbers left and those taken away together (that is A, B, C, D) compose an even Number.

PROP. XXIII.

Odd Numbers A, B, C , odd in Number, compose an odd Number.

Because an odd Number differs from A, B, C , an even Number by a Unite, $C - 1$ shall be an even Number; but the odd Numbers A, B compose an even Number; therefore $A, B, C - 1$ compose or make an even Number, but the sum of $A, B, C - 1$ differs from the Sum of A, B, C by a Unite; therefore the Sum of A, B, C is an e odd Number which W. Demonstrated.

Corollary.

IN like manner an odd being added to an even Number make an odd Number.

PROP. XXIV.

IF from an even Number D be Subtracted an even Number E , the remainder F , shall also be an even Number.

For

BOOK III.

of Arithmetick;

790

For if the remainder F were odd, with the even Number E , it would compose an odd Number; but F and E compose D ; therefore D would be odd, contrary to the Hypothesis.

D

E

$-$

F

A

Cor. Prop.

PROP. XXV.

IF from an even Number A , be Subtracted an odd Number B , the remainder C will be odd.

For if the remainder C were even, that with the odd Number B would make an odd Number; but C with B makes A ; therefore A would be odd, contrary to the Hypothesis.

A

B

$-$

C

Cor. 23.9

PROP. XXVI.

IF from an odd Number E , be Subtracted an odd Number F , the remainder G will be even.

For if G were odd, that with F would make E an even Number, contrary to the Hypothesis.

G

F

$-$

E

Cor. 23.9

PROP. XXVII.

IF from an odd Number A , be Subtracted an even Number B , the remainder C will be odd.

For if C were even, it with the even Number B , would compose an even Number A , contrary to the Hypothesis.

A

B

$-$

C

Cor. 23.9

G 3

PROP.

PROP. XXVIII.

AN even and an odd Number A, B , being multiplied together, produce an even Number C .

Def. 13

$A \ 2 \ B \ 3$
 $C \ 6$

$c \ 12. \ 9$

For C is b Composed of an even Number A , so often taken as there are Unites in B ; therefore C is an even Number.

Corollaries.

1. **A**N even Number multiplying an even Number, generates an even Number; this is manifest from Prop. 21.

2. Every evenly odd Number is even; this is manifest from Def. 19, Ax. 7. and this Prop.

PROP. XXIX.

AN odd Number D multiplying an odd Number E , produces an odd Number F .

Def. 13

For F is a composed of D , so often taken as there are Unites in the odd Number E , therefore F is odd.

$b \ 12 \ 9$

Corollaries.

1. **I**F an odd Number A , measures an even Number B , it measures the same by an even Number C .

Ax. 8
 $d \ 29. \ 9$

$12 \ B \}$
 $3 \ A \ } \ C \ A$

For if C were odd, because $A \times C = c \ B$, B would be an odd (being produced of an odd A , and an odd C) which is against the Hypothesis.

2. An odd Number D measuring an odd Number E , measures it by an odd Number F .

For

For if F were even, because D odd and F even, being multiplied, produce a E. F would be b even, contrary to the Hypothesis.

15 E 2 F 5 a Ax. 8
3 D 3 b 28 9

SCHOLIUM

From what is already said, it may be demonstrated, that no Number (A B) can be so cut or divided, that the Product of the whole (A B) multiplied into one part (C B) is equal to the Square of the other part A C; and consequently that the 11 Prop. 2 Book Eucl cannot be any ways applicable to Numbers, although the 10 First Propositions of that Book are also true in Numbers.

For if it be possible, let $AB \times CB = \square AC$. Now either AC is A — C — B even, and CB odd, or AC is odd, D — E — and CB even, or both AC and CB are odd, or both are even.

First, Let AC be even, and CB odd; therefore the a Cor. 23 9 whole AB is a odd; therefore $AB \times CB$ is b odd, but the b 29 9 Square of AC even, is c even, therefore the odd Number c Cor. 28 9 produced of $AB \times CB$, is equal to the even Number, to wit, the $\square$ of AC, which is absurd.

Secondly, Let AC be odd, and CB even; therefore the a Cor. 23 9 whole AB is d even; therefore $AB \times CB$ is e even, but e 28 9 the Square of the odd AC is f odd; therefore again this f 29 9 odd Number is equal to that even Number, which is absurd.

Thirdly, Let both AC and CB be odd, therefore the whole AB is g even; therefore the fact of AB even, in the g 22 9 odd CB is h even; but the Square of the odd AC i is odd, h 28 9 therefore again this odd Number is equal to that even number made of $AB \times CB$, which is absurd.

Lastly, let both AC and CB be even. Take two numbers, D, E, the least in the reason of AC to CB, then D, E are k prime the one to the other; and consequently both k 24 7

cannot be even, otherways a Binary would measure them, but either both are odd; or D is even and E odd; or D is odd and E even.

Now because $AB \times CB = \square AC$ it will be $AB :: AC :: AC :: CB$, and because $E :: D :: BC :: CA$ it will be by Composition $E + D :: D :: BA :: AC :: AC :: CB :: mD :: E$. Now because $D + E :: D :: D :: E$, therefore the fact of $D + E \times E = \square D$, which that it cannot be is Demonstrated in the three first parts.

1 shown before.
m Hyp.

But some perhaps will think that a Number may be cut or divided into two such parts as have Fractions belonging to them; but if those parts are reduced to two Fractions of the same Denomination, as shall be taught in Pract. Arith. Book. 2. Ch. 2. Prob. 2. it will easily appear that that also is impossible.

PROPOSITION XXX.

If an odd Number B measures an even Number A, it also measures its half.

Let the odd B measure the even A by C, then C is a even, and C shall also measure A by B; therefore also the half of the said C shall measure the half of A by B (for $C :: A :: \frac{1}{2}C :: \frac{1}{2}A$) therefore also B measures the half of A by the half of C which was to be Demonstrated.

a Cor. 9.
28. 9.
b Ax. 9.
c Ax. 9.

PROPOSITION XXXI.

If an odd Number A be Prime to any Number B, it shall also be Prime to C the double thereof.

For if A and C are not Prime to one another, let some Number D measure them, A by E, then D will of necessity be odd, otherways D even multiplied by E would produce A a even; wherefore because D odd measures C even (for C is even, seeing it is double of E) also D shall measure B the half thereof; but you suppose D measures A; therefore

a 28. 9.
b Prac.

fore A, B, are not Prime to one another, contrary to the Hypothesis.

Corollary.

AN odd Number A, which is Prime to any Number B is also Prime not only to the double of B, but also to its Quaduple and Octuple, &c. which is manifest from this Prop.

PROP. XXXII.

IF a double Proportion be continued from a Unite infinitely, it will only give all the evenly even Numbers.

That they are all evenly even is manifest from Prop. 1. A, B, C, D, E, F 1. Book 9. For by that 2 1. 2. 4. 8. 16. 32. 64 measures any in the Series, 1, R, P, Q by one of the Series, that is, by an even number; therefore by Def. 18.

That they are only evenly even is manifest from Prop. 13. Book 9. for seeing 2 is the next Prime to a Unite, no Number will measure any of them 2, 4, 8, &c. besides them which are in the Series, that is, besides the even Number; therefore by Def. 18.

That no Number besides these are evenly even only I thus shew; let there be only any evenly even Number O, whose half is P, this will be even, otherwise if P were odd, a Binary would measure O by the odd Number P, so O would be evenly odd contrary to the Hypothesis; therefore let R be the half of the said P; this again will be even, for if R were odd, seeing it is the fourth part of the said O, the Number 4 will measure O by the odd Number R contrary again to the Hypothesis; and so always the half of the preceding will be even, untill we come to Unity; therefore let 1 be the half of R, then R. P. O shall be doubled from Unity, and consequently

It shall be one of the Series, therefore the proposed is manifest.

PROP. XXXIII.

If all the odd Numbers be doubled, it will but produce all the evenly odd Numbers only (4. 6. 10, &c.)

a Hyp.

They are all evenly odd, because their half is a odd, therefore a Binary shall measure every one by a half odd Number.

3. 5. 7. 9. 11.

6. 10. 14. 18. 22.

A. B. C. D. E.

7 N 2

P Q

That they are only evenly odd, I thus shew; Take any of them, suppose D, whose half let be N, and set a Binary

b Def. 18. under it. If therefore D is also evenly even, *b P* even measures it by Q even; therefore because N *c Cor. 19. 7* also measures the same D by 2. it will be *c Q. N. 2. P.* but 2 measures the even Number P. therefore also Q even, measures N odd, which is absurd, and contrary to *Prop. 21. E. 9.*

d Cor. 28. 9 That none besides these are evenly odd only, I thus shew, if there are, they must be *d even*; but even Numbers either have even halves, and so are doubled from Unity, and therefore by the preceeding, they are only evenly even, or they have odd Halves, and so by the following they are evenly even, and evenly odd; therefore, &c.

PROP. XXXIV.

Even Numbers (A, B, C, D, E) which are neither doubled from two, nor have their half-parts odd, are evenly even, and evenly odd, and besides there are no other.

They are all evenly even, because that seeing their Halves

Halves are even, a Binary measures them by those even Halves; therefore by Definition 18. &c. And that they are also evenly odd, I thus shew: Take any of them as B, which if you divide into half, and the half again into half, and so on, at length you will happen on some odd Number (if we should always happen on even Numbers, at length we should come to 2, and so B would be doubled from Unity, contrary to the Hypothesis) which odd Number will measure the even Number B, by an *b* even Number, *b* Cor. 1. 29 therefore B is evenly odd.

Also from hence is manifest, the third part of the preceding Prop. And that besides these, there are no other evenly even, and evenly odd Numbers, is manifest from the two preceding Propositions.

PROP. XXXV.

IF there are Quantities, how many soever (EZ, CZ, BZ, AZ) in continual Proportion, then as CE, the excess of the second (CZ) above the first, or least EZ is to the least (EZ) so is the excess (AE) of the greatest (AZ) above the least (to all the rest) BZ, CZ, EZ, taken together; that is, $CZ - EZ = CE \therefore EZ :: AZ - EZ = AE \therefore BZ + CZ + EZ$.

Transfer BZ, CZ, EZ, into the greatest AZ, because $AZ \therefore BZ :: BZ \therefore CZ$

$:: CZ \therefore EZ$, therefore by a

Division, $AZ - BZ \therefore BZ ::$

$BZ - CZ \therefore CZ :: CZ - EZ$

$\therefore EZ$, that is, $AB \therefore BZ :: BC, CZ :: CE \therefore EZ$.

Wherefore as *b* $AB \therefore EZ$, or $CE \therefore AB + BC +$

$CE \therefore BZ + CZ + EZ$, which was to be Demonstrated.

$$\begin{array}{ccccccc} F & - & G & - & N & - & E - Z \\ & & & & & & C - Z \\ & & & & & & B - Z \\ A & - & B & - & C & - & E - Z \end{array}$$

a 17. 5

b 12. 5

From

From this Proposition, whose Facundity former Arithmeticians seem not to have observed, we have deduced the following

Corollaries.

1. **I**F there are Numbers how many soever, EZ, CZ, BZ, AZ , continually Proportional, then it will be, as the Denominator of the Proportion, or Ratio, less a Unit is to a Unit, so is the difference AE of the least and greatest terms, to the sum of all the terms less the greatest AZ .

Let FN . represent the Denominator, and GN the Unit; therefore $FN \dots GN :: CZ \dots EZ$, and by Division $FN - GN \dots GN :: CZ - EZ \dots FZ$, that is, $FG \dots GN :: CE \dots FZ$; but by this Prop. $CE \dots EZ :: AE \dots BZ + CZ + EZ$; therefore also $FG \dots GN :: AE \dots BZ + CZ + EZ$.

2. The same being posited, if the Denominator FN , and difference AE of the greatest and least Terms be given; the sum of all the Terms, except the greatest will be had, if the difference AE of the greatest and least be divided by FG , the Denominator less a Unit.

Cor. 1

Prac.

2 19 9

2 71

For seeing $c FG \dots GN$ a Unit : $AE \dots BZ + CZ + EZ =$ sum of all the Terms $- AZ$, it is exhibited if a fourth Proportional be found d and that is obtained; if the second Term, to wit, Unity be drawn in to the third, and the Product, which is still AE be divided by the first FG .

3. But if the Proportional Terms are Magnitudes, the sum of all less, the greatest will be had, if as the difference CB of two collateral Terms, is to the lesser EZ , so is AE the difference of the greatest and least Terms to another; this is manifest from the Prop.

4. If the given Series of proportionals whether Numbers or Magnitudes be in duplicate Proportion, the Summ of all the terms except the last will be had, if the least be taken from the greatest.

For by this 35 Prop. $CE \dots EZ :: AE \dots BZ \dagger CZ \dagger EZ$; but here $CE = EZ$ therefore also $AE = BZ \dagger CZ \dagger EZ$.

5. The same being posited, as the difference of the two greatest terms is to the greatest term, so is the greatest less the least to the Sum of all the terms less the least, that is, $AE = AZ - BZ \dots AZ :: AE = AZ - EZ = S - EZ$.

It is shewn in this Prop. $AB \dots BZ :: AE \dots BZ \dagger CZ \dagger EZ$; therefore $AB \dots AZ :: AE \dots AE \dagger BZ \dagger CZ \dagger EZ$, that is, all except the least EZ .

S C H O L I U M.

HE that compares this Proposition and its Corollaries, with what I have wrote in Prop. 11. Eucl. 6. will easily understand that from hence is deduc'd all that contemplation; whereby in a Series of Infinite Proportional quantities one is exhibited equal to all the Terms; for when the Number of Proportionals is finite, then $CE \dots$ *By this* EZ ; or $AB \dots BZ :: AE$ (that is the greatest less, the *35. Prop.* least or last) $\dots BZ \dagger CZ \dagger EZ$; but when the Number of Proportionals is infinite, then $AB \dots BZ ::$ greatest AZ whole $\dots BZ \dagger CZ \dagger EZ$, &c. where there is only this difference, that in the third place all the greatest Number AZ is taken, because the least which before was Subtracted from the greatest is now nothing, and so cannot be Subtracted.

Again when the Number of Proportionals is finite, $AB \dots g AZ :: AE \dots AE \dagger BZ \dagger CZ \dagger EZ$. But when *g Cor.* the Number of Proportionals is infinite, then $AB \dots AZ ::$ *Præc.* $AZ \dots AZ \dagger BZ \dagger CZ \dagger EZ$; where again, this is only the difference, because the least does now vanish and is nothing, and so cannot be Subtracted.

PROP.

PROP. XXVI.

IF a Series of Numbers continually Proportional from Unity in a duple Ratio (A, B, C, D) be continued until their Sum be a Prime Number E , the Sum being Multiplied into the greatest Term D shall produce a perfect Number.

Let there be taken so many Numbers E, O, N, P double from E as are the Numbers

a 22. 5.

1. $A, B, C, D.$ $A, B, C, D.$ by a equality; therefore $A \dots D :: E \dots P$; wherefore $A \times$

b 19. 7.

$M. Q.$ $P = b D \times E = f$; therefore P measures F by A a Binary, and consequently

c Cor. 4.

d Hyp.

$E, O, N, P.$ are continually Proportional in duple Ratio of the foregoing; therefore $F - E = e E + O + N + P$; but $E = d 1 + A + B + C + D$; therefore $F - E = 1 + A + B + C + D + O + N + P$. to each add E , then $F = 1 + A + B + C + D + E + O + N + P$, but the said Numbers are Aliquot parts of the said F , for seeing E, O, N, P, F , are continually Proportional, each measures $e F$, and for the same cause every one of 1. A, B, C , measures D , but D measures F , for $D \times f E = F$; therefore also every one of 1. A, B, C, D measures F .

e 11 9.

f Hyp.

g Ax. 12.

It remaineth to shew that there is no other aliquot part of the number F . Let M be any aliquot part of the said F , I will shew that it is the same with some of the aforesaid, A, B, C, D, E, O, N, P ; by the same reason it is denyed to be; for suppose M not to be the same with any of the said A, B, C, D, E, O, N, P . Then because M measures F let it measure it by Q therefore $M \times Q = b F$; but also $E \times D = F$; therefore $E \dots Q :: M \dots D$, but because B next to Unity is a Prime, to wit, $k 2$, and M is supposed to be diverse from all A, B, C ; M does not measure D , therefore neither shall E measure Q ; wherefore seeing $m E$ is a Prime, also EQ are n Prime one to the other, and therefore the o least in their Proportion. Therefore

b Ax. 8

k Hyp.

l 13. 9

m Hyp.

n 31. 7.

o 23 7.

E

E measures M, and Q measures D; therefore seeing p 21. 7. A is a Prime, Q is one q of the said A, B, C; there, q 13. 9. fore let Q be the same with B, and as many as are B, C, D, let so many be taken E, O, N, doubled from E, therefore by equality B . . D :: E . . N; therefore $B \times N = D \times E = F$. But because M measures F by Q; also $Q \times M = F$, wherefore seeing $Q \times M = F$ and $B \times N = F$, it follows $Q . . B :: N . . M$, but I have already shewn that Q is the same with B; therefore N is the same with M. Therefore F hath no other Aliquot parts besides 1. A, B, C, D, E, O, N, P. to all which because F is equal, it is a perfect Number, W. W. Demonstrated.

SCHOLIUM.

BY this Theorem may be found all the perfect Numbers; because the Sum of 1, and 2, is 3 a Prime Number, 3×2 makes 6 the first perfect Number, whose Aliquot parts are 1, 2, 3, and because the Sum of 1, 2, 4, is 7 a Prime Number, 7 multiplied by the greatest 4 makes 28 the second perfect Number, whose Aliquot parts are 1, 2, 4, 7, 14; again because the Sum of 1, 2, 4, 8, 16 is 31 a Prime, $31 \times 16 = 496$ the third perfect Number, whose Aliquot parts are, 1, 2, 4, 8, 16, 31, 62, 124, 248. The Sum will be easily had, if the following Number be lessened a Unite, as is manifest from Corollary 4. preceding, and the Aliquot parts of any perfect Number may be thus found; how many Numbers are taken doubled from Unity secluding Unity, take so many doubled from the Prime, or Sum numbering the Prime; these with those double from Unity, and the Unite its self are the Aliquot parts of the given perfect Number.

More.

Moreover, the manner of finding the perfect Numbers is briefly exhibited by this Proposition, and the 4 Preceding Corollary thus; let the Ratio of 1 to 2 be continued infinitely from a Unit; see what Numbers of the Progression by taking away a Unit make or become prime Numbers; for these being drawn into the next preceding them, will give the perfect Numbers.

Of the multitude of perfect Numbers already found, Mar. Mertennus in his Pref. Sect. 9. writes thus; That there are but 11 yet found, to wit, 6, 28, 496, 8,128, 550, 336, 8589, 869, 056, 137, 438, 691, 328, 2 385, 843, 008, 139, 952, 128. and three other, the last whereof is made of the 257th. Term of a duple Progression, lessened by a Unit, and multiplied into the 256 Term of those 28, which Peter Bungus, in the 28 Chap. of his Book of the Mysteries of Numbers rehearses; only the 8 first are perfect Numbers (to wit, those delivered above) the others are unperfect Numbers. Thus He.

But the Reason that more have not been yet found, is because that in a Series of a duple Progression the Intervals of the Numbers, which by lessening by a Unit become Primes, are so very great, for the 11th of the said Numbers is the 257th Term, from Unity exclusive (as is said above) and consequently the Number is very great, and the Labour to find whether or no it be a Prime, extream tedious; if the Number consists of 20 Figures, a whole Age is not sufficient to try it, let one use any way already found.

THE
FIRST BOOK
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PRACTICAL
Arithmetick.

Of whole Numbers.

IN every Number consisting of many Figures; that Figure which stands furthest towards the Right Hand, is said to be the First; and that which stands furthest towards the Left Hand is the Last.

CHAP. I.

Notation or the Institution of Arithmetical Figures.

THere are 10 Notes, or Characters, (some call them Digits, and some Figures) where by all Numbers are express'd, to wit,
0. 1. 2. 3. 4. 5. 6. 7. 8. 9.
The first Figure (1) signifies a Unite. (2) signifies two Unites. (3) three Unites, and so on. The last (0) (which is called a Cypher) signifies nothing, but
H being

being set before other Figures augments their value, as shall presently be shewn.

Besides this single value of a Note or Figure already spoken of, they have also another value, relating to the place every Figure is in, and the value of the places increase in a tenfold Proportion continued infinitely.

Therefore any Figure that stands in the first place signifies Unites, any Figure in the second place; so many Tens, as it hath Units; in the third place so many Hundreds; in the fourth so many Thousands; in the fifth so many Ten thousands; in the sixth so many Hundred thousands; in the seventh so many (Thousand thousands, or) Ten hundred thousands, or Millions, as the single Figure hath Units. After this manner, the value of the following places proceed in a decuple or tenfold Proportion infinitely.

As in the Number A, the first Figure (6) signifies six Units; the Second (9) is valued at nine tens of Units, that is, ninety (90) the third (7) seven hundreds 700. or seven hundred; the Fourth (5) five thousands 5000. The fifth (3) three tens of thousands, or thrice ten thousands, that is, 30000. thirty thousand; The Sixth (8) eight hundreds of thousands, that is 800000. eight hundred thousand; the Seventh (1) one million or one thousand thousands, or ten hundred thousands; the Eighth value (4) forty millions.

A, 4 1 8 3 5 7 9 6.

					9 0.	Ninery
					7 0 0.	Seven hundred
				5 0 0 0.	Five thousand	
			3 0 0 0 0.	Thirty thousand		
		8 0 0 0 0 0.	Eight hundred thousand			
					1 0 0 0 0 0 0 0.	a Thousand Thous. or Million.
					4 0 0 0 0 0 0 0.	Forty Million.

Unit

1 Unity
 10 Ten
 100 a Hundred
 1000 a Thousand
 10000 Ten thousand
 100000 a hundred thousand
 1000000 a Million.

Places.	VALUE.
1.	Units
2.	Tens
3.	Hundreds
4.	Thousands
5.	Tens of thousands
6.	Hundreds of thousands
7.	Millions
8.	Tens of millions
9.	Hundreds of millions
10.	Thousands of millions
11.	Ten thousands of millions
12.	Hundred thousands of millions
13.	Millions of millions or bilions
14.	Tens of bilions
15.	Hundreds of bilions
16.	Thousands of bilions
17.	Ten thousands of bilions
18.	Hundred thousands of bilions
19.	Millions of bilions or Trilions
20.	Tens of trilions
21.	Hundreds of trilions
22.	Thousands of trilions
23.	Ten thousands of trilions
24.	Hundred thousands of trilions
&c.	Millions of trilions or quatrillions, &c.

By this Table is shewn the value of all the places ; which I thought convenient to part thus ; that every six places may as it were constitute several Members : Therefore in the second Senary are Millions, in the third Bilions, in the fourth Trillions, in the fifth Quadrillions ; and so on infinitely : For the value of every Senary or Member, is denominated from the number of Senarys or Members going before it ; as if you desire to know the value of the seventh Member, it will be Sextillions, because six Senarys go before the seventh.

If you had rather express the value of the Members by Millions only, repeat the word *Million* so often as there are Members preceeding ; as if it were required to know the value of the fourth Senary, the word *Million* must be spoken thrice, because three Senaries preceed the fourth ; hence the value of the fourth Senary is Millions of Millions of Millions.

CHAP. II.

Numeration.

TEacheth to write and express any Number given, both which he may easily perform, that rightly understands both the simple and local values of Figures, explained in the foregoing Chapter ; but because in great Numbers, many (as well the skillful as others) are subject to stick ; there have been several ways invented to facilitate this work ; of which we will here deliver that which to us seems the most expedite.

First let the young Arithmetician endeavour readily to write and express the places of the first Senary or Member, that is, to write and express Numbers consisting of six, five, four and three Figures, &c.

In which he may be helpt by this Praxis.

Let the Numbers A, B, C, be given to be expressed; after the first three Figures put a Comma, that there may be as if it were two Members, whether the Second consists of three Figures or fewer: Then express the second Member as if it were alone, and to it add the word, *Thousand*, or *Thousands* once; but express the first Member as it lyes. Therefore you may express the three given Numbers thus;

A. Three hundred seventy Thousand, Five hundred and seventeen.

B. Forty Nine Thousand, Three Hundred and Four.

C. Seven Thousand, Fifty Nine.

This being premised, let any Number, ever so great, be given to be numbred, as;

D 96, 638 (908, 003 (030, 460 (243, 709.

After every six Figures beginning from the right Hand, make a lunula (, or a dash |, to divide the Number into several Members, every one of which will consist of six Notes, except the last, which may have less than six, even but one. Then express every Member, beginning at the left hand, as if they were alone, as has been already taught, but adjoin to every Member the value belonging to it, which the Table underneath shews.

Member.	Value.	Member.	Value.
First.	as it lyes.	Fourth.	Trilion.
Second.	Million.	Fifth.	Quatrilion.
Third.	Bilion.	Ec.	Ec.

Therefore the Number D must be thus expressed.

Mem^b. 4. Ninty Six Thousand, Six Hundred and Thirty Eight Trilions.

B 3

Mem^b.

Practical Arithmetick. BOOK I.

Mem. 3. Nine Hundred Eight Thousand and Three Billions.

Mem. 2. Thirty Thousand Four Hundred and Sixty Millions.

Mem. 1. Two Hundred Forty Three Thousand Seven Hundred and Nine.

When a Number hath many Cyphers uninterrupted towards the right Hand, the expressing thereof will be shorter; as if the Number E were given (which is greater than the Number of Sands contained in the Globe of the Earth.)

E 1 (000000 (000000 (000000 (000000 (000000 (000000.

Because its last (that is the seventh) Member consisting of a Unite is only Significant, you will express the whole Number by saying One Sextilion.

But if omitting the Words Billion, Trillion, Quadrilion, &c. you had rather express the Number by Millions only, having made the same Partitions as before, express every Member as if it were alone, adding the Word Million so often as there are other Members preceeding that which you are about to express, so the Number D will be expressed by Millions thus;

Mem. 4. 96. 638 Millions of Millions of Millions.

Mem. 3. 908. 003 Millions of Millions.

Mem. 2. 030. 460 Millions.

Mem. 1. 243. 709.

And the Number E will be expressed thus; One Million of Millions of Millions of Millions of Millions. The first way is shorter and most expedit, the other most used.

The easiest way of all, but far longer than the other, is to express Numbers by Thousands only, thus; After every three Figures beginning at the right hand

hand set a Comma; then express every Member, beginning at the left hand, as if they were alone, and add the Word Thousand or Thousands, so often as there are other Members preceeding that to be expressed. 1. Let the Number 8 be given to be expressed by Thousands; the last Member is 68 Thousand Thousand Thousands, the last but one, 638 Thousand Thousand Thousands.

F. 96, 638, 000, 000, 000.

The Reason of all which is manifest from the first Chapter.

The Writing of any Number is performed without any trouble, by help of the Table in the foregoing Chapter; as if this Number were given to be wrote in Figures, Forty Five Millions Seven Thousand. In the Table you will find Tens of Millions to be in the eighth place, Millions in the Seventh, and Thousands in the Fourth; therefore write 4 in the eighth place, 5 in the seventh, 7 in the Fourth, and fill up the vacant Places with Cyphers.

CHAP. III.

Porisms, on which the Reasons of Arithmetical Operations depend.

PORISM I.

IF one Figure be placed before a number (A) consisting of many Figures, its value is increased ten fold, if two, a hundred fold, if three a thousand fold; and so continually in a tenfold Proportion.

Demonstrat. For seeing that by placing one Figure before the number A, every Figure thereof is advanced a place higher, so that 3 standeth in the Second, 4 in the third, and 2 in the Fourth place; and thence the value of each Figure increases Ten-fold, as is manifest from the first Institution of Arithmetick, explained in the First Chapter; Therefore the value of all the whole number A is increased Ten-fold: Likewise if Two Figures be set before the Number A, every Figure thereof ascends two places higher; therefore the value of each Figure, and thence the value of all the Number is increased a Hundred Fold, or a Hundred Times, &c.

Corollary. Hence its manifest that if before Two Numbers a and b, are set equal Numbers of Cyphers to make them c and d; then c and d shall be Equi-multiplices of a and b: For if before each be placed one Cypher, then c and d shall be Ten-fold of a and b, or contain a and b ten times; if Two Cyphers a Hundred times, &c.

PORISM II.

O Two Numbers, that is the greatest which hath the most Figures in it.
 A 1000
 B 999
 This is manifest from the first Institution of Figures; so A is greater than B.

PORISM III.

O Numbers, containing equal Numbers of Figures, that is the greatest which hath the last Figure greatest.
 C 2000
 D 1999
 So C is greater than D, as is manifest from the first Institution of Figures.

P O

PORISM IV.

IF two unequal Numbers A, B, have equal Numbers of Figures, the greater does not contain the lesser Ten Times.

Demonstration. To the lesser Number B add a Cypher, and it makes the Number E; by the 2 Porism E is greater than A; but E contains the lesser Number B ten times, by Porism the First; Therefore A does not contain the lesser B ten Times, which was to be demonstrated.

A 2999
B 2000
E 2000|0

PORISM V.

THere are two Numbers given, whereof A is the greater, and B the lesser: The greater exceeds the lesser by one Figure; but the last Figure (3) of the lesser Number B, is greater than the last Figure of the greater Number A: I say the lesser is not contained ten times in the greater.

Demonstration. To the lesser Number B add a Cypher, and it will be E; by Porism 3, E is greater than A; but E containeth the lesser Number B precisely ten times by Porism II. Therefore the greater A does not contain the lesser B ten times. W W D.

A 2999
B 300
E 300|0

PO.

PORISM VI

TWO Numbers A and B are given; if the first Figure a of one of these Numbers multiplieth any Figure of a higher value, in the other Number as d, but taken simply; and the Product e be set in the place of the Figure d, and both so many Figures placed before it as are before d, so as to make f; I say that f is the true Product made by the Figure c multiplying d according to its local value.

3, 042 A
c
517 B

Demonstration. Let g be put for the Figure d, estimated according to the value of its place; then $g \times c = z$; now it is required to shew that $f = z$, because $c \times d = e$ and $c \times g = z$.

Therefore by Euclid. 17. 7. d. g. f. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100. 101. 102. 103. 104. 105. 106. 107. 108. 109. 110. 111. 112. 113. 114. 115. 116. 117. 118. 119. 120. 121. 122. 123. 124. 125. 126. 127. 128. 129. 130. 131. 132. 133. 134. 135. 136. 137. 138. 139. 140. 141. 142. 143. 144. 145. 146. 147. 148. 149. 150. 151. 152. 153. 154. 155. 156. 157. 158. 159. 160. 161. 162. 163. 164. 165. 166. 167. 168. 169. 170. 171. 172. 173. 174. 175. 176. 177. 178. 179. 180. 181. 182. 183. 184. 185. 186. 187. 188. 189. 190. 191. 192. 193. 194. 195. 196. 197. 198. 199. 200. 201. 202. 203. 204. 205. 206. 207. 208. 209. 210. 211. 212. 213. 214. 215. 216. 217. 218. 219. 220. 221. 222. 223. 224. 225. 226. 227. 228. 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1688. 1689. 1690. 1691. 1692. 1693. 1694. 1695. 1696. 1697. 1698. 1699. 1700. 1701. 1702. 1703. 1704. 1705. 1706. 1707. 1708. 1709. 1710. 1711. 1712. 1713. 1714. 1715. 1716. 1717. 1718. 1719. 1720. 1721. 1722. 1723. 1724. 1725. 1726. 1727. 1728. 1729. 1730. 1731. 1732. 1733. 1734. 1735. 1736. 1737. 1738. 1739. 1740. 1741. 1742. 1743. 1744. 1745. 1746. 1747. 1748. 1749. 1750. 1751. 1752. 1753. 1754. 1755. 1756. 1757. 1758. 1759. 1760. 1761. 1762. 1763. 1764. 1765. 1766. 1767. 1768. 1769. 1770. 1771. 1772. 1773. 1774. 1775. 1776. 1777. 1778. 1779. 1780. 1781. 1782. 1783. 1784. 1785. 1786. 1787. 1788. 1789. 1790. 1791. 1792. 1793. 1794. 1795. 1796. 1797. 1798. 1799. 1800. 1801. 1802. 1803. 1804. 1805. 1806. 1807. 1808. 1809. 1810. 1811. 1812. 1813. 1814. 1815. 1816. 1817. 1818. 1819. 1820. 1821. 1822. 1823. 1824. 1825. 1826. 1827. 1828. 1829. 1830. 1831. 1832. 1833. 1834. 1835. 1836. 1837. 1838. 1839. 1840. 1841. 1842. 1843. 1844. 1845. 1846. 1847. 1848. 1849. 1850. 1851. 1852. 1853. 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2020. 2021. 2022. 2023. 2024. 2025. 2026. 2027. 2028. 2029. 2030. 2031. 2032. 2033. 2034. 2035. 2036. 2037. 2038. 2039. 2040. 2041. 2042. 2043. 2044. 2045. 2046. 2047. 2048. 2049. 2050. 2051. 2052. 2053. 2054. 2055. 2056. 2057. 2058. 2059. 2060. 2061. 2062. 2063. 2064. 2065. 2066. 2067. 2068. 2069. 2070. 2071. 2072. 2073. 2074. 2075. 2076. 2077. 2078. 2079. 2080. 2081. 2082. 2083. 2084. 2085. 2086. 2087. 2088. 2089. 2090. 2091. 2092. 2093. 2094. 2095. 2096. 2097. 2098. 2099. 2100. 2101. 2102. 2103. 2104. 2105. 2106. 2107. 2108. 2109. 2110. 2111. 2112. 2113. 2114. 2115. 2116. 2117. 2118. 2119. 2120. 2121. 2122. 2123. 2124. 2125. 2126. 2127. 2128. 2129. 2130. 2131. 2132. 2133. 2134. 2135. 2136. 2137. 2138. 2139. 2140. 2141. 2142. 2143. 2144. 2145. 2146. 2147. 2148. 2149. 2150. 2151. 2152. 2153. 2154. 2155. 2156. 2157. 2158. 2159. 2160. 2161. 2162. 2163. 2164. 2165. 2166. 2167. 2168. 2169. 2170. 2171. 2172. 2173. 2174. 2175. 2176. 2177. 2178. 2179. 2180. 218

Demonstration. Let h . d
 and g . be put for the Fi- 3. 0420 A
 gures, k and d estimated k
 according to the value of 3. 17. B
 their places, then $h \times g$ —————
 = z ; now it is required c. 15. m. 15. 000
 to shew that $f = z$; before f. 15. 000. 00
 c. the Product of $k \times d$ put —————
 so many Figures as are be- k. 51. 15. 00. b
 fore the Figure d , so as to g. 3. 000
 make m , by the precedent c. 15. 000. 15. 000. 00
 Porism m shall be the Pro- m. 15. 000. 15. 000. 00
 duct of k multiplied into z. —————
 the Figure d estimated according to its places value,
 that is $m = k \times d$; Now because by Construction m
 = c with so many Figures placed before it as are in
 the number A before d , and by Hypothesis f is the
 same with c , but with so many Figures placed before
 it, as in the numbers A and B are together before d
 and k ; it's manifest that the Figures which in f are put
 before c , exceed those which in m are put before c ,
 by so many Figures as are in B before k , that is, by so
 many Figures as are in h before k , seeing h is the same
 with k estimated according to its local value: There-
 fore by Corollary to Porism 2. h and f are equi-multi-
 plices of k and m , and thence $k : h :: m : f$; and be-
 cause as above $k \times g = m$, and by Hypothesis $h \times g =$
 z ; therefore by Eucl. 18. 7. $k : h :: m : z$. wherefore
 $m : f :: m : z$, therefore by Eucl. 9. 5. $f = z$; which
 was to be demonstrated.

From this and the foregoing Porism, Multiplication
 is demonstrated in Chap. 7.

PORISM VIII.

A B (60 C
 360,5897 (6 d
 (6,6000 f

L Et there be two Numbers AB, and C. given; if C divideth the Member A simply taken; and the quotient d be advanced to the place of the Member A, that is, if before d there be placed so many Figures as are before the Member A, as f; I say that f is the true quotient; which is had by dividing the Member A, estimated according to the value of its place, by the Divisor C.

Demonstration: Let g, be put for the Member A estimated according to the value of its place; then C dividing g, quotes z . it is required to shew that $f = z$. because C dividing A quotes d and C dividing g quotes z therefore by Ax. 8. Eucl. 7. $A = C \times d$ and $g = C \times z$ therefore by Eucl. 18. 7. $A \dots g :: d \dots z$, also $A \dots g :: d \dots f$ (for seeing g and f are the same with the Numbers A and d, encreased to equal places, that is, having by Hypothesis an equal number of Figures placed before them, it is manifest from Corol. porism 2. that g and f are equimultiples of A and d) hence $d \dots f :: d \dots z$; therefore by Eucl. 9. 5. $f = z$ W. W.

From this Porism Division is Demonstrated, Chap. 9.

PORISM. IX.

THe Artifice of Arithmetical operations, that is of Addition, Substraction, Multiplication and Division, consists herein; that while we work, neglecting the value of places; Figures are taken only according to their simple value. Nevertheless, so as the Sums in Addition, the remainders in Substraction, the products in Multiplication, and the Quotients in Division are set in their right places; whence it follows that every Figure hath its due value according to its place.

PORISM X.

One single Figure multiplying or dividing any Number, encreases or lessens the same but by one Figure or place

Let there be given any Number, as *A*, this multiplied by 10 is encreased but by one Figure, Degree, or place higher than it was at first, as is manifest from Porism I. Therefore a number multiplied by a single Figure is not encreased more than one place higher, and the lesser the Multiplier is, the lesser is the Product.

Again, if 10 divides *A*, it shall not lessen it more than one Degree or place, Therefore a single Figure, &c. and the lesser the Divisor is, the greater the Quotient shall be.

$$\begin{array}{r} A. 365 \\ 10 \\ \hline 3650 \end{array}$$

$$A \begin{array}{r} 365 \\ 10 \end{array} \left(\begin{array}{r} 36 \\ 5 \\ \hline 10 \end{array} \right)$$

CHAP. IV.

Addition:

Addition is the Collection of many Numbers into one Sum.

OPERATION.

9706 3. A
800 2. B
504 1. C

L Et the numbers A B C. be given to be added together, or Collected into one Sum.

Write the Numbers so under one another, that the first Figure may be under the first (that is, Units under Units) the second under the second (that is, Tens under Tens) the third under the third (that is, Hundreds under Hundreds) and so on.

Then having drawn a line under them, add the Figures of the first place together; and if the number composed of these consist but of one Figure, write it under the line in the first place; but if it consist of two Figures, the first of them only must be wrote under the line, and the other must be reserved to be placed under the next place; if of three which seldom happens, the second belongs to the second place, and the third to the third.

Next add together the Figures of the second place with that which was reserved, and write the number composed of them under the line in the second place, minding the caution already given: after the same manner the Addition of the remaining places is performed.

These precepts will be made more plain by an Example.

Ex-

Example.

The Figures of the first place, 1, 2, 3, make 6, which I write under the Line; the Figures of the Second place, 4, 0, 6, make 10, which number because it consists of two Figures, I write the first Figure 0, under the Line; the second 1 I reserve to be placed under the third place. In the third place I find not any significant Figure, wherefore I write the reserved Figure 1 under the Line in the third place. In the fourth place I find 5, 8, 7, which together make 20, which Number consisting of Two Figures, I write the first 0 under the Line in the fourth place; but reserve the second Figure 2, for the next place. In the fifth place I find 9, to which I add the reserved Figure 2, and they make 11, which I write in the fifth and sixth place as they lye; and thus the Operation being ended, the Number D is got, which is the Sum of the given Number A, B, C.

If there be many ranks of Numbers to be added together, it will be best to divide them into three or four Classes, by putting lines between them, and to get the Sums of every Class, which being afterwards brought to one, will give the Sum of all the Sums.

Demonstration.

The Demonstration is gathered from Porism 9 Chap. 3. for although in Arithmetical Operations, the local value of the Figures is neglected for a time, yet it was restored, while the Sums of every place were set in their places, as is manifest from the Operation it self.

of

Of divers Species.

IF divers Species or Numbers of several Names, as Pounds, Shillings and Pence are to be added together, write like Species, or numbers of one Name, one under another, and the least, to

L.	s.	d.	
9	17	6	wit, Pence in the first place, and the other in order; then out of the sum of the lesser Species, which is 19 Pence,
8	04	3	take the next Species, to wit, a Shilling so often as you can, and write the
5	02	2	remainder below the Line under Pence,
3	04	8	and reserve the 1 Shilling taken out,
			to be added to the next Species, that is,
26	08	7	to Shillings. The next Species being
			added together make 27 Shillings, to

which adding 1 Shilling taken out of the first Species the Sum will be 28 Shillings, out of this may be taken 1 Pound, which I add to the next Species, that is, to the Pounds, and the remaining 8 Shillings I write under the Shillings. The Sum of the third Species is 25 pound, to which add 1 taken out of the foregoing Species and the Sum will be 26 Pound, which write under the Pounds, and so the Sum will be 26 Pound 8 Shilling and 7 Pence.

After the same manner, numbers of any other Species may be added together, as is done in the following Examples,

Example 1.

L.	oun.	dr.	gr.
23	07	16	13
47	10	15	07
16	04	11	08
15	05	11	11
24	11	18	12
98	4	13	3

Example 2.

hun.	hun.	qu.	l.	oun.	dr.
12	11	3	18	11	13
12	10	2	15	10	10
12	09	1	12	10	14
9	09	3	13	11	17
9	08	1	10	10	10
56	10	0	15	7	12

Ells

Examp. 3.

Tds. qu. na.

24 : 3 : 2

34 : 3 : 1

21 : 2 : 2

19 : 1 : 3

100 : 3 : 0

Examp. 4.

Ells qu. na.

15 : 3 : 2

16 : 4 : 3

19 : 2 : 1

23 : 3 : 2

65 : 4 : 0

Having thus shewn how to add together several Integral Numbers, and also Numbers of diverse Species, or Denominations, it remains for the better helping the young Learner to give some account of the several Divisions, the Money, Weight, and Measure of this Nation are divided into; and how many of the lesser Species is contained in the next greater, the knowing whereof will be also very necessary, both in Multiplication and Division, for the reducing of the greater Species into the lesser, and the lesser into the greater, and will be of general use throughout all this Treatise of *Practicable Arithmetick*.

The most usual Coins used in *England*, are Pounds, Shillings, Pence, and Farthings; whereof 4 Farthings make a Penny, 12 Pence a Shilling, and 20 Shillings a Pound Sterling, Charactered thus, *l. s. d.* *Money*

There are three Sorts of Weight commonly used in *England*,

1. *Troy Weight*, used in the weighing of Bread, *Troy weight* Gold, Silver, &c. the usual Species of which are, Pounds, Ounces, Penny-weights and Grains, whereof 24 Grains make a Penny-weight, 20 Penny-weights make an Ounce, and 12 Ounces a Pound, Charactered thus, *l. oz. pw. gr.*

Averdupois 2. *Averdupois Weight*, which is sub-divided into little and great.

Averdupois little weights. *Averdupois little Weight*, is used in the weighing of Sugar, Pepper, Cloves, &c. by the Retail, the usual Species of which are Pounds, Ounces, and Drams, whereof 8 Drams make an Ounce, and 16 Ounces a Pound; Charactered thus, *ls oz. dr.*

Averdupois great Weight. *Averdupois great Weight*, is used in the weighing all Commodities that are sold by the Hundred, as Currants, Wool, &c. the usual Species of which are Tuns, Hundreds, Quarterns of a Hundred, Pounds and Ounces; whereof 16 Ounces make a Pound, 28 Pounds a Quarter of a Hundred, 4 Quarters, or 112 Pound an Hundred Weight, 20 Hundred Weight a Tun, Charactered thus, *T. C. gr.*

Apothecaries Weight. *Apothecaries Weight*, Divide into Grains, Scruples, Drams and Ounces; whereof 12 Grains make a Scruple, 3 Scruples a Dram, 8 Drams an Ounce, and 12 Ounces a Pound; Charactered thus, *gr. dr. oz. lb.*

These are several Sorts of Measures used in England, some Liquid, some Dry, and some relating to the measure of Time and Motion, of which take as follows.

Liquid Measure. 1. *Liquid Measure*, used in the measuring of all Liquid Substances, the several Species whereof are, Tuns, Pipes, Hogheads, Gallons, Pottles, Quarts, and Pints; of which 2 Pints make a Quart, 2 Quarts a Pottle, 2 Pottles a Gallon, 63 Gallons a Hoghead, and 2 Hogheads a Pipe, 2 Pipes or Butts a Tun; Charactered thus, *T. P. hbg. gal. pottle. q. pint.*

Dry Measure. 2. *Dry Measure*, used in the measuring of all Dry Substances, as Corn, Salt, Coals, Sand, &c. the several Species whereof are Weys, Chaldrons, Quarters, Bushels, Pecks, Gallons, Pottles, Quarts, and Pints, whereof 2 Pints make a Quart, 2 Quarts a Pottle, 2 Pottles a Gallon, 2 Gallons a Peck, 4 Pecks a Bushel Land Measure, 5 Pecks a Bushel Water measure, 8 Bushels a Quarter, 4 Quarters a Chaldron, and 5 Quar-

Quarters a Wey, Charactered thus, *W. Ch. Qr. Bush.*

Pec. Gall. Pott. Qr. Pt.

3. *Long Measure*, used in the Measuring of Land, *Long Mea-*
Board, Glass, &c. the several Species whereof are, *sure.*
Miles, Furlongs, Perches, Feet, Inches, and Barley-
Corns; of which 3 Barley-Corns make an Inch, 12
Inches a Foot, 3 Feet a Yard, 5 1/2 Yards, or 16 1/2
Feet, a Pole or Perch, 4 Perches a Furlong, 8 Fur-
longs an English Mile.

4. *Cloth Measure*, used in the Measuring of Wool-Cloth *Cloth Mea-*
len, and Linnen Cloth, &c. the several Species *sure.*
whereof are, Yards or Ells, Quarters and Nails;
whereof 4 Nails make a Quarter, 4 Quarters a Yard,
and 3 Quarters an Ell English, 3 Quarters an Ell
Flemish; Charactered thus, Yds. Ells. Qrs. N.

5. *Time*, consisteth of Years, Months, Weeks, *Time.*
Days, Hours and Minutes; whereof 60 Minutes make
an Hour, 24 Hours a Day, 7 Days a Week, 4 Weeks
of 28 Days, a Month, 12 Months a Lunar Year.

CHAP. V.

Subtraction

TEacheth to take or subtract a lesser Num-
ber from a greater, and to shew the
remainder, which may be done two
ways.

Practice of Method 1.

Write the lesser Number under the greater, so
as the first Figure may Answer to the
first, the second to the second, the third to the
third, &c.

I 2

Then

Then draw a line under the two Numbers, and take the first Figure of the lower Number from the first of the upper one, and write the remainder under the line in the first place; then take the second from the second, and write the remainder in the second place, and so proceed: but if in the upper Number there remain any Figures, which none in the lower Number answer to, write them also under the line.

If any one of the inferior Figures be greater than the upper, to the uppermost add 10 in your mind; and estimate the next significant Figure, to the left hand in the upper Number, less by a Unite than it is; and if there be one or more Cyphers between Significant Figures, they are all to be reckoned Nines.

Examples.

496 A 1. Let B be given to be Subtracted
 75 B from A; take 5 out of 6, and there re-
 ——— mains 1, which write under the Line;
 421 C then take 7 out of 9, and there remains
 2, which write under the Line in the
 second Place; now there is left in the upper Num-
 ber the Figure 4, to which no Figure in the lower
 Number answereth, therefore write 4 in the third
 place; and the Remainder sought is C.

2. Let E be given to be Subtracted from D; be-
 cause 8 cannot be taken out of 4, I add
 3004 D 10 in my Mind to 4; and then take 8
 2598 E out of 14 and there remain 6, which I
 ——— write under the Line; but because I ad-
 .426 F ded 10 in my Mind to 4, which was
 borrowed from the following Number,
 we must reckon the next Significant Figure 3, only
 as 2, and the intermediate Cyphers as so many Nines;
 therefore I take 7 out of 9, and write the remainder
 2 under the Line; also I take 5 out of 9, and write
 the Remainder 4 under the Line; lastly, I take 2,
 not

not out of 3, but out of 2, and there remains nothing; therefore the Remainder sought is F.

3. Let H be given to be Subtracted from G, because 9 cannot be taken out of 3, adding 10 in mind to 3, I take 9 from 13, and write down the Remainder; wherefore 5 is made 4, and 00 are turned into 99; therefore 6 out of 9 and there remains 3, which I write down; then because there are no more Figures of the lower Number H to be Subtracted, I write the remaining Figures of the upper Number, which now are not 850, but 849 under the Line, and I shall be the remainder sought.

$$\begin{array}{r} 85003 \text{ G} \\ 69 \text{ H} \\ \hline 84934 \end{array}$$

4. Let it be required to Subtract L from M, because 3 cannot be taken from 0, adding in Mind 10 to 0, I take 3 out of 10 and there remains 7; then because in the Number K the next significant Number is 1, that vanisheth, and every Cypher is turned into 9, from which Subtract, as above.

$$\begin{array}{r} 100000 \text{ K} \\ 51243 \text{ L} \\ \hline 48757 \text{ M} \end{array}$$

Demonstration of Method 1.

It is manifest from the Operation it self, that in writing down the Remainders, each Figure keeps its local Value; what remains to Demonstrate is, why, when the lower Figure cannot be taken out of the upper, the next significant Figure towards the left hand is lessened by a Unit, and the intermediate Cyphers changed every one into 9.

See the second Example D E F, because 8 cannot be taken out of 4, to 4 I add 10, borrowed from the following Number, which is 3 Thousand; but it is plain, that if from three Thousand I take

$$\begin{array}{r} 3004 \text{ D} \\ 2578 \text{ E} \\ \hline 426 \text{ F} \end{array}$$

I 3

ten,

ten, there will remain two Thousand nine Hundred and Ninety; now because they are expressed in Figures thus, 2990, its manifest why in the upper Number, 3 is turned into 2, and the Cyphers 00 into 99.

But if any one should say, if before the first Figure 4 of the Number D, be set two other Figures 96, and before the first 8 of the Number E, two, 91, then 4 will stand for 400, and 8 for 800, and then to Subtract 800 from 400, there must be added not 10 but a 1000, that so 800 may be taken from 1400; I answer, that it will come to the same thing, whether you take 8 out of 14, or 800 out of 1400, because the Remainder 6 is written under the Line, in the same place which the Figures 4 and 8 belong to, to wit, in the third; whence it comes to pass that the Remainder 6 being set in that place, standeth for 600, that is, so much as would remain, if 800 were taken from 1400.

Practice of Method 2.

This differs from the first only in this, that when the lower Figure cannot be taken from the upper, and therefore ten is added to it, there is no alteration made in the next upper Figures following; but the following inferiour Figure is increased by a Unit, or if no significant Figure follow, then One or a Unite must be supposed to be set in the following place; this will be more plain by an Example.

Example.

Let L be given to be Subtracted out of K, 6 from 8 and there remains 2, 7 from 6 I cannot, therefore

fore I add ten, then 7 from 16, and there remains 9; now because I added 10, the next Interior Figure 5 is to be increased a Unit, and so 5 becomes 6; again 6 from 0 I cannot, therefore I take 6 from 10, and there remains 4; but because I cannot add 10, I suppose a Unit to be set in the next place, which is vacant, which Subtracted from 8 leaves 7, therefore the remainder sought is M.

This Method for the most part is more expeditious than the former, yet that will be the most convenient, when in the greater Number many Cyphers follow one another.

Demonstration of Method 2.

Let there be given two Quantities, AB the greater, and CD the lesser; Increase the said Quantities by the equal Quantities EA and IC. then if you take I—C—F IF out of EB, the remainder LB shall be the same, as if CF was taken out of AB; this if applied to Numbers, the reason of the Operation will presently appear; for to add 10 to the upper Figure 6, so as to make it 16, is nothing else than to put a Unit after it in the next place (which here is the third) wherefore seeing after the Interior Figure 7, is also put a Unit, in the next place (which is also here the third) it is manifest that each Number is equally increased, and so the Remainder got after this manner, is the same which remains if the Number L, be taken from the Number K.

Diverse Species

If diverse Species or Numbers of several Names, as Pounds, Shillings, and Pence, be given to be Sub-

tracted one from another, write the lesser Sum under the greater, so as like may answer to like, and if like cannot be taken from like, borrow One, or a Unit from the next greater Species.

Example.

$ \begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 25 : 12 : 10 \\ 10 : 14 : 11 \\ \hline 14 : 17 : 11 \\ \hline \end{array} $	Because 11 d. cannot be taken out of 10 d. from the 12 s. I borrow one that is 12 d. and add it to 10 d. which makes 22 d. then 11 out of 22, and there remains 11; and again, because I cannot take 14 out of 11, (having taken away one before) to 11 I add one Pound or 20 Shillings, borrowed from the 25 £. which makes 31; now 14 from 31 and there remains 17. Lastly I subtract 10 £. from 24 £. (for I was borrowed before) and there remains 14; so the remainder sought is 14 £. 17 s. 11 d.
--	--

Or thus.

Because I cannot take 11 from 10 d., I borrow one Shilling or 12 d. which added to 10 makes 22, then 11 out of 22 and there remains 11, which done, I carry 1 that I borrowed to the next Species, and add it to 14, then 15 from 12 I cannot; therefore borrow 1 pound or 20 Shillings which added to 12 makes 32, now 15 from 32 and there remains 17. Lastly I carry 1 that I borrow'd to pounds which add to 10 makes 11, then 11 from 25 and there remains 14; so will be had the remainder 14. 17. 11. the same with the former.

After the same manner any other Species may be Subtracted, as in the following Examples.

l.	ox.	pw.	gr.	C.	qr.	l.	oz.	dr.
423	: 10	: 12	: 11		431	: 3	: 17	: 11
324	: 11	: 11	: 14		249	: 2	: 18	: 15
98	: 11	: 0	: 21		82	: 0	: 26	: 11

Ells qr. n.

341 : 1 : 2

121 : 3 : 2

219 : 3 : 0

yds. qr. n.

213 : 2 : 2

198 : 3 : 3

14 : 2 : 3

Questions to Exercise Addition and Subtraction.

Question 1. **W**HAT Number is that which being Subtracted from 4898 the remainder is 329. *Answer 4569.*

From 4898 Subtract 329, and there will remain 4569, which is the Number sought, for that being taken from 4898 there remains 329 as was required.

$$\begin{array}{r} 4898 \\ - 329 \\ \hline 4569 \end{array}$$

Question 2. What Number is that to which if three times 486 be added, the Sum will be 10000. *Answer 8542.*

$$\begin{array}{r} 4898 \\ - 4569 \\ \hline 329 \end{array}$$

Question 3. What Number is that, from which if 3489 be Subtracted, the remainder will be 4689. *Answer 8178.*

Question 4. Since Queen Elizabeth began her Reign, is 138 years in what year of our Lord was it. *Answer 1550.*

Question 5. A Man says that this present year 1688, he is 89 years of Age, what year was he born in. *Answer 1599.*

Quest.

Question 6. Two persons A and B are of several Ages, the Age of the Elder is 70 years, and the difference of their Age is 20, what is the Age of B?

Answer. 50 years.

Question 7. A lent B, 345 *l.* 14 *s.* 6 *d.* and after some time B repays A, 40 *l.* 13 *s.* 8 *d.* at one time, and 100 *l.* at another, and gives him a Bill to receive 29 *l.* 10 *s.* how much has B. paid A. in all, and what remains yet due to the said A.

Answer. Paid in all : 230 : 13 : 1

Unpaid : 215 : 10 : 10

Question 8. A Linnen Draper sold 6 pieces of Muslins, each piece contained 24 Ells, 3 quarters and 2 Nails, and was to have 5 *l.* 18 *s.* 8 *d.* per piece, how many Ells &c. were there in all, and what were they all sold for. *Answer* 149 Ells¹ and sold for 35 *l.* 12 *s.*

Question 9. A man pays away a certain Sum of money and forgets to enter it into his Cash Book, till a day or two after; at which time he forgets what the Sum was; but recollecting himself, he remembers that if to the Sum were added 3 Guineas, it would make a 100 *l.* so desires to know, from a young Scholar what the Sum was he paid away, it is required to know what Answer he will give. *Answer* 95 *l.* 14 *s.*

Quest. 10. A Grocer buys 4 Hogsheds of Sugar, each Hogshhead weighed Gross, 19 C. 3 qr. 18 *l.* and the Tare of each was 24 *l.* what was the neat weight of all the Hogs-heads. *Answer.*

To resolve this Question, you must know that Gross weight is the weight of the Sugar and Cask taken together; Tare is the weight allowed for the Casks only, and the neat weight that of the Sugar alone; the like must be understood of all Garbelled Commodities.

Hence, the Question must be resolved thus, first write the Gross weight of the 4 Hogshheads under one another, and get their Sum; then also get the Tare of all the Hogshheads, and Subtract the Tare from the Gross, and the remainder shall be the neat weight required.

A D C

1	2	3	4	5	6	7	8	9	10	11	12
11	12	13	14	15	16	17	18	19	20	21	22
23	24	25	26	27	28	29	30	31	32	33	34
35	36	37	38	39	40	41	42	43	44	45	46
47	48	49	50	51	52	53	54	55	56	57	58
59	60	61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80	81	82
83	84	85	86	87	88	89	90	91	92	93	94
95	96	97	98	99	100	101	102	103	104	105	106

1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81	82	83	84
85	86	87	88	89	90	91	92	93	94	95	96
97	98	99	100	101	102	103	104	105	106	107	108

B



1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81	82	83	84
85	86	87	88	89	90	91	92	93	94	95	96
97	98	99	100	101	102	103	104	105	106	107	108

1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81	82	83	84
85	86	87	88	89	90	91	92	93	94	95	96
97	98	99	100	101	102	103	104	105	106	107	108

THE ROYAL MAIL

A D										C									
1	2	3	4	5	6	7	8	9											
2	4	6	8	10	12	14	16	18											
3	6	9	12	15	18	21	24	27											
4	8	12	16	20	24	28	32	36											
5	10	15	20	25	30	35	40	45											
6	12	18	24	30	36	42	48	54											
7	14	21	28	35	42	49	56	63											
8	16	24	32	40	48	56	64	72											
9	18	27	36	45	54	63	72	81											

A D	
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18

B E

X

1	5	9	7	
2	10	18	14	1194
3	15	27	21	1791
4	20	36	28	2388
5	25	45	35	2985
6	30	54	42	3582
7	35	63	49	4179
8	40	72	56	4776
9	45	81	63	5373

B E

Sqr Cube

0	1
0	4
0	9
1	6
2	5
3	6
4	9
6	4
8	1

The Table set with Rods on it

CHAP. VI.

Of the Pythagorian Table, serving for Multiplication and Division.

THE Pythagorian Table (which is so called from *Pythagoras*, who is said to be its Author) hath on the top, or in the uppermost Rank A C, the 9 Original Arithmetical Figures, 1, 2, 3, 4, 5, 6, 7, 8, 9 and under every one of them is set its Double, Triple, Quadruple, &c. to its Nonuple; the use whereof is very great, and is shewn in Chap. 7. and 9.

Now if the Columns AB and DE &c. of this Table be separated one from another, so as that they may be mixed amongst themselves, or set together in any order; thence will proceed an admirable compendium, for Multiplication and Division; the Lord *John Neper* Baron of *Merchiston*, was the first who Invented and Published this Section of the Pythagorian Table; to whom both for this Invention, and that of the Logarithms, the Mathematical World are infinitely indebted.

The construction of this Dissected and moveable Table, is after this manner; let there be prepared of Brass, thick Paper, or any other fit matter, several oblong and thin Rods as AB, which divide into nine equal Squares, and cut each Square except the uppermost, into two Triangles by drawing Diameters; so as the lower Triangle B N P may be towards the right hand; on one side of the first Rod write the first Column of the Pythagorian Table, to wit, 1, 2, 3, 4, 5, 6, 7, 8, 9, and on the other side, the second Column 2, 4, 6, 8, &c. On

one side of the Second Rod write the third Column of *Pythagoras's* Table, 3, 6, 9, 12. &c. and on the other side the fourth Column, 4, 8, 12, 16, &c. and so on the other Rods, write the remaining Columns of *Pythagoras's* Table; always observing that when the Number consists of but one Figure, it must be wrote in the lower Triangle; if of two as 18, then the first Figure 8 must be wrote in the lower, and the second 1 in the upper Triangle, By each of these Rods, more may be made of the same kind and form; but 6 or a fort are enough to perform the greatest Multiplications and Divisions; besides these there is one Rod more as XZ to be prepared; which is seen by the Figures. This last is always set to the left hand in every Multiplication and Division, shewing what Rank each Number is in; which from its office, is called the exponent.

But for the orderly laying and keeping the Rods in their due position; some have contrived a little Table or Frame, a little broader than the length of the Rods, and about once and a half the length of the breadth; upon the superficies whereof close to one of the Edges is glewed, or otherwise fastened the said exponent Rod; under the end of which is also fastened a ledge the length of the Frame or Table; which when thus made and finished, is called a *Tabeller*, and is moved by X, Z.

The chief and principal Artifice of these Rods, and whence all the other Uses spring, is, that they shew in almost a moment, any Number Multiplied by the 9 Original Figures, as if the Number 397 were given; let those Rods that will make up this Number in their tops, be set together on the *Tabeller* aforesaid. So the exponent Rod XZ will stand to the left hand; and the Number 397 will stand in the first or uppermost Rank, and under it eight other Ranks of Numbers, which are such Multiplices of the first 397 (that is, contain 397 so many times), as the Number on the exponent denotes, as is Demonstrated Chap. 8.

How

*How to Read and Write out any Number,
contained in any Rank on the Rods.*

IV. CHAP.

TO do this, observe three things. 1. That the first lower Triangle which is towards the right hand makes the first place, every intermediate Rhombus, every intermediate place; and the last upper Triangle, the last place; from which it will presently appear how many Figures a Number consists of, which is contained in any Rank on the Rods. 2. That the Figures which stand in one and the same Rhombus (as in N M) must be added together: But if when added together, they make a Number consisting of two Figures, then the second Figure must be added to the following Rhombus or Triangle. 3. That if

To perform the Multiplication of Numbers, you must first know the Multiplication of single Figures.

Example. Let the 9th Rank be given to write out. In the first inferior Triangle I find three, which I write in the first place; in the following Rhombus, the Figures are 6 and 1, which added together make 7; therefore I write 7 in the second place, in the following Rhombus are the Figures 8 and 5; which added together make 13, I write the first Figure 3 in the third place; the other 1 I carry to the following last Triangle, and add it to the Number found there, which is 4, and they make 5, which I write in the fourth place; therefore the ninth Rank hath in it the Number 3735.

The Reason of writing out the Numbers from the Rods thus, shall be shown in the Demonstration of Chap. 8.

FIGURE of the lower Number B, and write down the first Figure towards the right hand, from the right hand towards the left, observing that if the Product consists of two Figures, to write down the first only, the other keep in mind, and add to the next Product.

CHAP. VII.

Multiplication.

ONE Number is said to Multiply another, when the Number Multiplied is so often added to it self, as there are Units in the Number Multiplying; and another Number is produced; Or thus, one Number is said to Multiply another, when the Number that is produced is to the Number Multiplied, as the Multiplier is to a Unit.

To perform the Multiplication of Numbers, consisting of many Figures expeditiously; you must readily know the Multiplication of single Figures, which by the help of *Pythagoras's Table*, is done in a moment; as if it were required to Multiply 7 by 8, in the side AB seek 7; in the side AC 8; in the angle of their meeting is found 56, which is the product of 7 by 8.

Practice.

LET there be given two Numbers A, B, to be Multiplied one by another; if one of these Numbers consists of more Figures than the other; write that which hath fewest undermost; then Multiply every Figure of the upper Number A, by the first Figure of the lower Number B, and write the Products under the line, from the right hand towards the left; observing, that if the Product consists of two Figures, to write down the first only; the other keep in mind, and add to the next Product.

In

In like manner the other Figures of the lower Number B, must multiply every Figure of the upper Number A; but the Products must be written so, as the first Figure D of the second Product may stand in that place, which the Multiplying Figure is in, that is, in the second; the first E of the third Product, in that place which the Multiplying Figure 3 is in, that is in the third place, &c.

Then Collect the Products, C, D, E, into one Sum F, which shall be the Product sought, by the Multiplication of the Numbers A, B.

Example.

Let the Numbers A and B be
 given to be multiplied: First multiply the Number A by 7, the first Figure of the lesser Number B. then 7 times 2 makes 14; the first Figure whereof 4 I write under the line, the other to wit, 1, I keep in mind, 7 by 4 makes 28, to which I add the 1 kept in mind, and it makes 29, I write 9 in the second place under the line and reserve 2; 7 times 0 makes 0, therefore I only write the reserved Figure 2, in the third place; 7 times 3 make 21, which I write in the fourth and fifth place, as they follow.

3042 A
 717 B

21294 C
 3042 D
 21000 E

After the same manner the Number A is Multiplied by 1, the second Figure of the Number B, producing the Product D, the first Figure 2 of which, is wrote in the second place, that is, under its Multiplying Figure which is 1; and so also the Number A is Multiplied by 5, the third Figure of the Number B, and produces E, the first Figure 0 of which is wrote in the third place, that is under its Multiplying Figure 5, but the other are placed in order, in their following places.

R

Demon



Demonstration.
It is manifest from the 6 and 7 Porisms of Chap. 3.

that all the Figures in the first Product C, have their value due to their respective places, also that the first Figure of the Product D, and the first Figure of the Product E have the like, is manifest from the 6 Porism; and that the remaining Figures of the Products D and E, have their due value according to their respective places, is manifest from Porism the 7th. all which will be very plain, if the Operation already prescribed be compared with the forementioned Porisms.

Compendiums of Multiplication.

012000 G
400 H

4800000 K

23 . . . G
3000 . . H

115009 K

2340 M

1000 N

2340000 P

8012 Q

5000 R

48078 S

4006 T

40113078

1. **W**hen the first Figures of the given Numbers G H, are one or more Cyphers following one another, the Multiplication is performed only between the significant Figures, and the Cyphers of each Number, being taken together, are set before that Product.

2. When one of the given Numbers as M, consists of Unity and Cyphers, you have the Product P, if to the other Number N be added all the Cyphers of the first M.

3. If in the Multiplier R, one or more Cyphers following one another, are included between significant Figures, then neglecting the Cyphers or Cypher; Multiply the Number Q by the following significant Figure S, and write the first Figure.

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gure of the Product T under its Multiplying Figure, which is here 3, and is in the fourth place.

The Demonstration of these *Compendiums*, is also manifest from 6 and 7 *Porisms* of Chap. 3.

Corollary.

TO Multiplication belongeth that part of Arithmetick, wherein any Number of a higher Species being given, it is required to find how many of a lesser Species are contained therein, which is called Reduction Descending, because in Operation we go from a higher to a lower Species.

Practice.

TO Reduce Numbers of a higher Species to a lesser, Multiply the higher Number given by so many of the lesser Species, as are contained in one of the greater, and the Product shall be the Number required; as if it were required to know how many Shillings are in 30 l. you must Multiply the said Sum by so many Shillings as are in one pound, to wit, by 20 and the Product is 600 Shillings.

If it were required to know how many Pence there is in 489 pound.

Multiply 489 by 20 to reduce 489
it first to Shillings, then Multiply 20
the Shillings by 12, because 12 d.
makes one Shilling, and the Product
will be 117360 the Pence con-
tained in the Number given.

If it were required to know how many pence there is in 48 l. 19 s. 10 d.

$$\begin{array}{r}
 489 \\
 \times 20 \\
 \hline
 9780 \\
 \times 12 \\
 \hline
 117360
 \end{array}$$

Multiply

4

48 : 19 : 10

20

979

12

1958

980

11758

Multiply the Pounds as before by 20, and to the Product add the odd 19 s. which makes 979 Shillings, Multiply this by 12, and to the Product add the odd 10 d. which makes 11758, which is the pence contained in the Number given.

After the same manner, any other higher Species may be reduced to Species of a lower kind.

The Demonstration of which is gathered from what is already delivered, and from the Definition of Multiplication.

SCHOLIUM

TO Multiply from the left Hand to the right, is of great use in Division; and is performed in this manner. 1. No Figure is reserved or kept in mind, which is very convenient. 2. When the last Figures are Multiplied together, if by Multiplying the rest, any Product happen to be two Figures, the Figure on the left hand must be wrote under that Figure which in the foregoing Product is the first, or on the right hand.

Example.

183

6

688

41

1098

If 183 is given to be Multiplied by 6, then 6 times 1 is 6; therefore I write 6 under the line, 6 times 8 is 48, because this Product consists of two Figures, I write the left hand Figure 4 under 6 and 8 before 6; 6 times 3 is 18. I write 1 under 8, and 8 before 8, which being added together, the Product sought is 1098: The Demonstration is gathered from what hath been already said.

CHAP. VIII.

Multiplication by the Rods.

MULTIPLICATION is performed with great expedition, by the Rods of *Pythagoras's Table*; the construction of which is shewn in *Chap. 6.* Now the excellency and great use shall be declared.

Practice.

Let the Numbers A and B be given to be Multiplied together. 1. Seek those Rods which have the Figures of the Number A at the top, which place so together that the Number A may stand in the upper Rank, and before them set the exponent Rod XZ. 2. Then on the exponent Rod XZ seek 8 the first Figure of B the lesser Number given, and write out the Rank answering thereto; to wit, the eighth, which set under the Line, and will be C; after the same manner, seek 4 the second Figure of the Number B, on the exponent Rod, and write out the Rank answering thereto, to wit, the Fourth, which will be D, whose first Figure 8 must be wrote under the second Figure of the first Product C, if the Number B had more Figures, they likewise being found on the exponent Rod would shew the other Ranks of Figures to be wrote out. 3. Lastly the Products, C, D being added together, shall give the Product of A by B sought.

597 A

48 B

4776 C

2388 D

28656 E

See the
Table
Chap. 6.



Demonstration.

The Demonstration is easie both from the construction of the Rods delivered in Chap. 6. and from what has been Demonstrated of common Multiplication, for the Number 56 found in the Eighth Square of the first Rod, is 8 times 7, and got by Multiplying 7 by 8; but because 56 consists of two Figures, I write 6 the first Figure in the lower Triangle under the line, and reserve 5 the other Figure in the upper Triangle; also the Number 72 found in the Eighth Square of the second Rod, is 8 times 9, that is, 9 Multiplied by 8; but because 72 consists of two Figures, the second 2 belongs to the third place, but the first 7 to the second only, and the reserved Figure 5 belongs also to the same place; wherefore 5 and 2 must be added together; the Sum which is 7. write in the second place under the line, which is the reason why the Figure in the upper Triangle, must be always added to the Figure of the following Triangle; also why the Figures in the same Rhombus, belong to the same place, at least as to the first Figure of their Sum, for if the Collected Figures of any Rhombus make a Number consisting of two Figures; the second Figure is carried to the following Rhombus. Lastly the Number 40, found in the 8 Square of the third Rod, is made of 5 Multiplied by 8, whose first Figure 0 only belongs to the third place; to which because the reserved Figure 7 also belongs, standing in the same Rhombus it must be added; but because 0 added to 7 makes 7, write 7 in the third place, and the other, which is 4, and in the upper Triangle, write in the fourth place; therefore the Eighth Rank on the Rods, that is, the Number C, is that which is made by Multiplying A. 597 by 8 the first Figure of the Number B. By the same reason may be shewn that the fourth Rank, that is, the Number

Number D, is that which was made by Multiplying A 597 by 4, the second Figure of B, therefore their Sum E is the Number which is made by Multiplying A by B, W. W. D.

Hence the chief Artifice of these Rods consists in two things: First, That the Rods may be so composed and put together that any Number may stand in the upper Rank. Secondly, That Numbers consisting of two Figures, are so placed on the Rods, that the first Figure stands in the lower Triangle, and the other in the upper; whence it happens that because every Superior Triangle with the Inferior triangle of the next following Rod, constitutes a Rhombus, the upper Figure of the first Rod (which is always that which is kept in mind) is found in the same Rhombus, with the first or lower figure of the following Rod, with which it must be added, to be wrote in the same place with it.

Moreover how expedite, easie, and secure this method is, any that are willing to experience it, will soon find by use.

CHAP. IX.

Division.

ONE Number is said to divide another, when the Number found or Quotient shews how often the Divisor is contained in the Dividend; or thus, one Number is said to divide another, when the Number or Quotient is to a Unit, as the Dividend to the Divisor.

I shall give two different Methods of dividing, whereof the first is least used, but is the best; the other most used, but the difficultest.

Practise of Method 1.

Let the Number A C, be given to be divided by the Number Z.

$$\begin{array}{r}
 \text{Z} \overline{) \text{A B C}} \text{ (X} \\
 597 \overline{) 45993} \text{ (769} \\
 \underline{4179} \\
 \text{D } 411, 9 \\
 \underline{368, 2} \\
 \text{E } 537, 3 \\
 \underline{537, 3} \\
 0
 \end{array}$$

1. Out of the Dividend take so many of the last Figures as are contained in the Divisor; or if these make a Number less than the Divisor (as it happens in our Example, seeing 459 is less than Z, 597) put on more to it as 0; and so the first Member A B of the Division is made, which distinguish by a point, and call the Dividual.

2. See how often the Divisor Z is contained in the Member A B, or Dividual; if it do not appear plain, seek how often the last Figure of the Divisor is contained in the last two Figures of the Member A B, (but if the Divisor and Dividual consist of equal Number of Figures, then must be sought how often the last Figure is contained in the last of the Dividual) which you will find to be contained 7 times; therefore write 7 after the crooked line, or Lunula; but you must never write more than 9 at once, after the said Lunula, because, as I will Demonstrate below, the Divisor is never contained in the Dividual more than 9 times.

3. Multiply all the Divisor Z by (7) the Figure already wrote in the Quotient, from the left to the right Hand; the Product 4179, which must be either less than, or equal to the Dividual, write under the said Dividual and subtract it from the same, which done, the remainder is D, 411, which must be less than the Divisor Z, and write it under the line placing a point before the first Figure thereof; and if all this succeeds the

Fi.

Figure 7 wrote in the Quotient after the Lunula shall be the right Figure, and the Division of the first Member performed.

4. But if the Figure wrote in the Quotient, when it Multiplies the Divisor, exhibits a Product greater than the Dividual, reject that, and take a less in its stead; but if it gives a Product too small, that is such as when you have subtracted it from the Dividual, it gives a remainder greater than the Divisor; rejecting that also, a greater must be taken; which Tryal of the Figure to be set after the Lunula, is soonest performed by Multiplying from the left hand, towards the Right, and is shewn in the fore-going Chap.

Wherefore the Difficulty of Division chiefly consists here-in, to write such a Figure in the Quotient, by which if the Divisor be Multiplied, the Product may be subtracted from the Dividual, and the remainder be either nothing, or less than the Divisor, for such a Figure will shew how often the Divisor is contained in the Dividual.

5. The Division of the other Members is performed after the same manner, to the right hand of the remainder D (411) of the first Member AB, set the Figure 9 which goeth before the first Member AB; and so the second Member or Dividual is 411, 9 the Operation of which must be like the first. viz. Seek how often the Divisor Z is contained therein, which you will find six times; then write 6 after the Lunula before 7, and Multiply the Divisor by 6, and set the Product 3582 under the dividual 411, 9; then Subtract one from the other, and place the remainder E 537, under the line, and set a point before it.

Lastly, To the remainder E, 537, write the Figure 3 which follows to the right Hand, and you have the third Member 537, 3. Again seek how often the Divisor is contained in the third Member or Dividual, which you will find nine times, therefore set 9 in the Quotient, and Multiply the Divisor by 9, the

the Product is 5373 which Subtract from the Member 5373 and there remains nothing.

This being done all the Division of the Number given AC by Z is ended.

6. If when the Division is ended, any thing remains (as in this Example there remains 5) write the remainder 5 over the Divisor 13, and draw a line between them, making the Fraction N; then the Integer placed after the Lunula, together with the Fraction N, shall be the Quotient.

7. If when the first Operation be done, it happens, that the Divisor 3 is not contained once in any of the other Members or Dividuals, (as 0. 4) then set a Cypher to the Quotient after the Lunula, and to that Member or Dividual set one Figure more of the Dividend, (as 5) a small dash being put between, that it may appear that the Member (0. 4 | 5) is made up of two, that so the Number of Dividuals may appear.

8. To divide a lesser Number by a greater as 3 by 7, the lesser must be wrote over the greater, drawing a line between them, to make the Fraction R, which is Demonstrated 2 Book, 2 Chap. 1 Theor. 2 Corol.

$$\frac{3}{7} R$$

Demon-

4

Demonstration.

There are so many Figures wrote after the Lunula, and so the Quotient X hath so many Figures, as there are Members or Dividuals in the Division, as is manifest from the foregoing Operation; but there are so many Members as there are Figures preceding the first Member AB, in the Dividend AC, which is manifest in its self, therefore the Quotient X consists of so many Figures, as go before the first Member of the Dividend: Wherefore it is manifest from Porism 8, Chap. 3. that the last Figure 7 wrote after the Lunula of the Number X, is the true Quotient of the first Member AB; that is, which tells how often the Divisor Z is contained in the first Member AB; in like manner the same is shewn of the other Figures of the Number X wrote after the Lunula in respect of the rest of the Members or Dividuals, therefore the Number X is the true Quotient procured by Dividing the Number AC by Z. W. W. D.

If the Divisor does not measure the Dividend, the true Quotient nevertheless is an Integer (39) with the Fraction (N) whose Numerator is what remains after the Division is ended, and its Denominator the Divisor its self which I thus Demonstrate.

$$\begin{array}{r}
 \text{Divisor} \\
 13 \overline{) 51.2(39\frac{1}{3}N} \\
 \underline{39} \\
 12.2 \\
 \underline{117} \\
 5
 \end{array}$$

At
/

As a Unit is to the integral part of the Quotient; so is the Divisor to the Dividend less, the last remainder, that is, $1 \dots 39 \div 13 \dots 512 - 5$; from the *I. Theor. Chap. 2. Book 2.* as a Unit is to a Fraction, so is the Denominator to the Numerator; that is, $1 \dots N = \frac{1}{3} \div 13 \dots 5$, by *Euch. 14. 5.* as a Unit is to the Integer and Fraction, - so is the Divisor to the Dividend; that is $1 \dots 39 \frac{2}{3} \div 13 \dots 512$: Wherefore according to the Definition of Division $39 \frac{2}{3}$ is the true Quotient; which was to be Demonstrated.

Why more than 9 must never be wrote at once in the Quotient, is thus shewn; Either the Member or Dividual A, and Divisor B, consist of equal Number of Figures, or the Dividual exceeds the

A. 2996 Divisor by one Figure, for it never ex-

B. 1000 ceeds it by more, as is manifest from the first Part: If therefore they have equal

Number of Figures, it is apparent from *Porism 4. Chap. 3.* that B is contained less than ten times in

A; but if the Member or Dividual A, exceed the Divisor B by one Figure, then the last Figure of the

Divisor is greater than the last Figure of the Dividual, as is manifest from the first Part; and therefore

also by *Porism 9. Chap. 3.* B is contained

A. 2999 less than ten times in A; wherefore seeing

B. 309 the Divisor is contained in every Dividual less than ten times, the reason is

manifest why we must never write more than 9 at once in the Quotient.

Practice of Method 2.

Let the Number A be given to be Divided by the Number B.

1. Write the Divisor under the Dividend towards the left hand, so as the last Figures, and last but one, &c. of each, may answer one another; but if the Number 370, which is over the Division, be less than the same, as in *Schem. 1.* then the Divisor must

must be removed one place forwarder to the right hand, as is seen in the Example. But whatsoever is towards the left hand of the Divisor, is reckoned as placed over the Divisor, whence 3702 is over 821, and 37 is over 8, and 570 is over 2.

Schem. 1

A 37029 (4
B 821

Schem. 2

5
37029 (4
821

Schem. 3

4
52
37029 (4
821

Schem. 4

41
528
37029 (4
821

2. See how often the Divisor is contained in the Figures over it; but because it is very difficult for the most part to judg of this, see how often the last Figure 8 of the Divisor, is contained in the Number 37, wrote over it, and you will find it to be 4 times, therefore write 4 after the Lunula.

3. Multiply all the Divisor by the Figure 4 wrote in the Quotient, and Subtract the Product from the Figures 3702, which are wrote over the Divisor; then write the remainder over the foregoing Figures, and Cancel the Divisor and the Figures which were over it; but most choose to Subtract every Figure of the Divisor multiplyed separately by the Quotient 4, from the Numbers over them; that is, they multiply every Figure of the Divisor distinct by the Quotient, and Subtract every particular Product of such Multiplication from the Figures which are over it.

Example



Example.

- a Sch. 1.* *a* 8 By 4 makes 32, which Subtracted from 37;
b Sch. 2. there remains 5, Cancel *b* 37 and 8, write 5 over
 7; again 2 by 4 makes 8, which Subtracted from
c Sch. 3. 50 leaves 42, Cancel *c* 50, and 2 in the Divisor, and
 write the remainder over 50; lastly 1 by 4 makes 4,
 which Subtracted from 422, there remains 418;
d Sch. 4. then Cancel *d* 422 and 1, and write the remainder
 418 over 422; and thus one application of the Di-
 visor is performed.

4. But because 8, the last Figure of the Divisor is multiplied by the Quotient 4, and the Product Subtracted from 37; and that the last Figure but one (2) is also multiplied by the same Quotient 4, and the Product Subtracted from the remainder 50 placed over it; and likewise, that the first Figure 1, multiply'd by the same Quotient (4) is also Subtracted from the Remainder 422 placed over it; therefore hence it is manifest, that when it is sought how often the last Figure 8 of the Divisor, is contained in the Figures 37 standing over it, such a Figure must be placed after the Lunula in the Quotient, by which all the Figures of the Divisor being multiplied, the Product may be Subtracted from the Figures wrote over the Divisor, and yet under this further Restriction that the last Subtraction being performed, the Remainder (418) may be either nothing, or less than the Divisor, as is explained in *Se^t. 4. of Method 1.* Wherefore, if when the first Subtraction is ended, the second cannot be performed; or if when these are done, the third cannot, the Figure wrote after the Lunula must be lessened by Unity, so often till every one of the Subtractions may be performed; therefore before you Work in earnest, all the Operation in *Se^t. 3.* must be wrought mentally, that the Figure wrote in the
 Quo.

Quotient may be tryed, whether it be not too little, or too great.

We will explain what hath been said above, by an Example.

Let the Number M be given to be divided by N;

I seek how often N is contained in the Number 738 placed after it, which I find to be 3 times, therefore I write 3 after the Lunula; then 2 by 3 makes 6, which taken from 7, there remains 1; 3 by 4 makes 12, which taken from 13, there remains 1; 9 by 3 makes 27, which cannot be taken from 18, therefore the Figure 3 wrote in the Quotient is too big; wherefore rejecting that, in its place set 2, and repeat the tryal; 2 by 2 makes 4, which Subtracted from 7, there remains 3; 4 by 2 makes 8, which Subtracted from 33, there remains 25; 9 by 2 is 18, which Subtracted from 238, there remains 240, therefore the right Figure is 2.

Sch. 5.

21

M 7382(32

N 249

Sch. 6

24

350

7382(2

249

But although most mix alternately Subtraction with Multiplication, as is above explained; yet to me it seems the more adviseable, to perform all the Multiplication together, and that also from the left hand to the right, always remembering the caution prescribed in Sect. 4. Method 1.

5. The

5. The first Application of the Divisor being performed, as is shewn in *Sche. 1, 2,*

Sch. 7.

41

338

37029 (4

821

3, 4. remove the Divisor B one Figure towards the right hand, as you see done in *Sch. 7*, and the manner of working thorow all will be the same as was delivered before; and if there are more Applications to be made, they are to be performed after the same manner.

6. If when the Divisor is removed, it cannot be contained once in the Figures placed over it, then write a Cypher in the Quotient, and remove the Divisor a place more forward, not Cancelling any Figure in the Dividend.

Sch. 8.

42

3284

A 37029 (45

B 8211

82

7. And if any thing remains when the Division is ended, do therewith as is Prescribed in *Self. 6. Method 1.*

In *Sch. 8.* the whole Division of the Number A by B is exhibited, which before, or above, was exhibited in several *Sch.* only to make it

the more clear and apparent.

The Demonstration of this Method, is like the foregoing Demonstration.

But although this last way of Division is for the most part used every where, yet I esteem the first way to be much the better; for the second way Cancels all the Figures so, that it confounds the steps and members of the Division; so that if there should be any Error, it would be very difficult to Correct it, as is sufficiently apparent from *Sch. 8.* but the first sheweth distinctly every Member of the Division, the Product of every Multiplication, and

what

what remains in every Subtraction of the Products from each Member ; whence the amendment is easie, if any error has happened.

The Use of the common Table of Pythagoras in Division.

LET there be given any Number consisting of two Places, as E, and any single Figure, as F, and let it be required to find how often F is contained in E.

Seek the Figure F in the side A B, and in the Numerical Line answering it, seek the Number E, or the next less than it, as (56) from this Number ascending to the side A C, you find the Quotient sought, as 7.

The reason hereof is manifest from the Construction of the Table in Chap. 6.

62 E See Table
8 F in Chap. 6.

Compendiums in Division.

1: WHEN the last Figure of the Divisor D is a Unit, and all the rest Cyphers. From the Dividend C take so many Figures (829) as the Divisor hath Cyphers, and write those which are left (2889), after the Lunula, to which set the Figures cut off 829 wrote over the Divisor 1000, drawing a Line between them, and the whole Number 2889, with the Fraction $\frac{829}{1000}$ shall be the Quotient. But if the Figures which are cut off are also Cyphers, as in this Example, then those Figures only which remain to the left hand shall be the Quotient, to wit, 342.

C 2889 | 829
D 1000
G (2889 $\frac{829}{1000}$)

L

D

Demonstration.

This is easily gathered from *Porism 8. Chap. 3.* and is also manifest from the second *Compend. in Multiplication, Chap. 7.* For 1000 multiplying 342 | 000 342, produceth 342000; therefore 1000 Divif. a 1000 dividing 342000 quotes 342. (342 for from the Def. of Multiplication, Quotient. Chap. 8. $1 \dots 1000 :: 342 \dots 342000$. Wherefore by permutation, $342 \dots 1 :: 342000 \dots 1000$. Therefore by the Def. of Division; 1000 dividing 342000, gives 342 for the Quotient.

2. When the Divisor consists of one or more significant Figures, other than a Unit, and hath Cyphers to the right hand, from the Dividend A cut off so many Figures 46 to the right hand, as the Divisor hath Cyphers; and divide the remaining Figures 249, only by the significant Figures of the Divisor; if when the Division is, ended any thing remains, as here there remains 1, set that with 46 cut off, over the Divisor, drawing a Line between, which place after the whole Number 124 in the Quotient; then the whole Number 124, with the Fraction M shall be the Quotient.

$$\begin{array}{r}
 \text{A } 2, 4, 9 \mid 46 \\
 \underline{2} \qquad \text{B } 2 \mid 00 \\
 9. 4 \\
 \underline{} \\
 0. 9 \text{ (124 } \frac{246}{2} \text{ M} \\
 \underline{8} \\
 1
 \end{array}$$

The Demonstration hereof is deduced from *Porism 8. Chap. 3.* or from the first *Compendium in Multiplication Chap. 7.* by the like reasoning as above.

3. When

3. When the Dividend hath Cyphers to the right hand, and you have wrought till none of the significant Figures remains, then write so many Cyphers in the Quotient, as there are Cyphers left in the Dividend, after the last Member or Dividual.

Let C be divided by D, 4 in the first Member 27 is contained 6 times; therefore I write 6 after the Divisor, 4 by 6 is 24, which I write under the Dividual 27, and being subtracted therefrom, there remains 3, which I write under the Line; and so I write the next Figure 2 of the Dividend; that I may have the second Member 32, 4 in 32 is contained 8 times; I write 8 in the Quotient, 4 by 8 makes 32, which subtracted from 32, there remains nothing; and because in the Dividend there remains nothing but Cyphers, I write so many in the Quotient, and the Division is ended, which is Demonstrated, as before.

$$\begin{array}{r}
 27 \overline{) 272} \quad 68000 \\
 \underline{24} \\
 32 \\
 \underline{32} \\
 0
 \end{array}
 \quad D \ 4$$

CHAP. X.

Division is performed very easily, by the Rods of Pythagoras his Table.

IN Division the greatest trouble is to find the Quotient by which the Divisor is to be Multiplied; which Quotient, as also the Product of the Divisor multiplied by the Quotient, the Rods do exhibit in an excellent Compendium: The rest of the Division is performed by the first Method in the precedent Chapter, which suits these Rods far better, than the second in the said Chapter.

Let the Number AC be given to be divided by the Number Z.

$$\begin{array}{r}
 \text{Z} \) \ \text{A B C} \quad \text{X} \\
 597 \) \ 4590, 9, 3 \quad 769 \\
 \underline{4179} \\
 \text{D} \ 411, 9 \\
 \underline{358, 2} \\
 \text{E} \ 537, 3 \\
 \underline{537, 3} \\
 0
 \end{array}$$

1. Set the Rods together, so as they may have the Divisor Z in their first and uppermost rank; besides the Scheme placed here, view the Table Z X, in Chap. 6

2. Determine the first member A B, as in Sect. 1. Method 1. Chap. 9.

3. Seek what rank on the Rods, hath a number equal to the Dividual, or that is next less (but that rank is next less than the Dividual, that hath immediately following it a rank greater than the Dividual) and the number on the Exponent Rod denominating or pointing out the rank, is the Quotient, which write after the Lunula.

In our Example you will find the Rank next less than the Dividual to be the Seventh, therefore write 7 after the Lunula.

4. Write the Rank found out (which is the product of the Divisor by the Quotient 7) under the Member or Dividual A B, and being subtracted therefrom, write the remainder (411) under the Line, to which set the Figure 9, that goeth before the first Member AB in the Dividend, and so you have the second Member (411.9.)

5. The Division of the second Member, and of all the following, must be performed after the same manner as the first.

6. If the Member or Dividual be less than the Divisor, that is, than the first Rank on the Rods, you must write a Cypher in the Quotient; and to the Dividual must be placed one Figure more of the Dividend

vidend, and so you have a new Member or Dividual.

7. If the Ninth Rank be less than the Dividual, write 9 in the Quotient.

Finally, by a very small Exercise, you will perceive in a moment which Rank on the Rods is next less or greater than the Dividual; also the 2 and 3d. *Porisms* in Chap. 3. are to be called to Mind, as likewise what is before taught, in relation to the writing out, and reading the Ranks; but you must chiefly consider the last quadrate of the Ranks, to wit, how great the Figure is which is set in the last Triangle to the left hand, and whether the Sum of the Figures in the following Rhombus, exceeds 9. and so whether any Figure is to be carried to the said last Triangle.

Demonstration.

The Demonstration is gathered from what is Demonstrated in Chap. 9. and from the Definition of Division.

But none knows how excellent this way of Division is, but they who have experienced it; for by this one may perform more Divisions in an Hour or two, than in a whole Day by the common way; to which Compendium both of Time and Operation, you have also the security of working, without being subject to mistake.

Corollary.

To Division belongs that part of Arithmetick, wherein any Number of a lower Species being given, it is required to find how many of a greater Species are contained therein, which is called *Reduction ascending*, because in Operation we go from a lower to a higher Species.

Practice.

To reduce Numbers of a lesser Species to a greater, divide the given Number by so many of the lesser Species as are contained in one of the greater, and the Quotient is the required.

Example.

20) 428/8 (214

4

02

2

08

8

0

12) 433 (36

36

73

72

1

If it were required to know how many Pounds there are in 4280 Shillings.

Divide the Shillings by 20, because 20 Shillings make a Pound; and the Quotient shall be 214, the Pounds required.

It is required to know how many Pounds are in 433 Pence.

Divide the Pence first by 12, which quotes 36 Shillings, which Divide again by 20, and it will quote 1, the Pounds required, and 16 Shillings 1 Penny over.

The remainders of each Division are the odd Shillings and Pence over and above the Pounds, contained in the Number given,

and are denominated from the Dividend, of which they are part; the like must be understood of other Species.

It is required to know how many Hundred weight are contained in 48978 Ounces.

Divide the Ounces by 16, the ounces in 1 pound, so you have 30601 l which Divide again by 28, the pounds in one Quarter of an Hundred; so you have 1092 qrs. of a C. which divide again by 4, the qrs. in an Hundred, and you have 273 Hundred: hence the Answer is 273 Hund. o qrs. 25 l. 2 oz.

$$\begin{array}{r}
 \text{oz. 28) l.} \\
 16 \overline{) 48978} \quad (30601 \quad (1092 \\
 \underline{48} \quad \quad \quad 28 \\
 097 \quad \quad \quad 260 \\
 \underline{96} \quad \quad \quad 252 \\
 18 \quad \quad \quad 81 \\
 \underline{16} \quad \quad \quad 46 \\
 2 \quad \quad \quad 25 \\
 4 \overline{) 1092} \quad (273 \\
 \underline{8} \\
 29 \\
 \underline{28} \\
 12 \\
 \underline{12} \\
 0
 \end{array}$$

Questions to Exercise Multiplication and Division.

Q. 1. **W**Hat Number is that which being Divided by 1423 quotes 48? *Answer,*

Resolved by Multiplying 1423 by 48.

Quest. 2. What Number is that, by which if you divide 4834, the Quotient will be 342? *Answer.*

Resolved by Dividing 4834 by 342.

Q. 3. What Number is that, by which if you multiply 342, the Product will be 34878? *Answer.*

Q. 4. What Number is that, which multiplied by 232, the Product will be 4764? *Answer.*

Q. 5. The Circumference of the Earth is estimated 360 Degrees (60 miles making a Degree) how many Barley Corns are contained therein?

L 4

Q. 6.

Q. 6. From London to York, is 2861200 Barley Corns; How many Miles is it? *Answer.*

Q. 7. A Gentleman gave 300 Purles of Money to his Daughters Portion; in every one of half the Purles, was 100 broad Pieces of Gold, valued at 2s. In every one of the other half, was 200 Crowns, and 50 Guineas, English Coin; How many Pound Sterling was her Portion valued at? *Answer.*

Q. 8. A House was Built and Finished the first day of March, in the Year of our Lord 1620; How many Minutes has it stood to this present 14 day of July, 1688? *Answer.*

CHAP. XI.

The Proofs of Addition, Subtraction, Multiplication, and Division.

THE safest way is to prove these several Rules one by another, for all other ways are subject to Error.

To Prove Addition.

Addition is proved by Subtraction:

Let X be the Sum made by the Addition, of the Numbers A and B; subtract either of the Numbers added, suppose B, from the Sum X, if the remainder C is the same with the other A, the Addition is right; if not there is an Error.

123	A
25	B
148	X
25	
123	C

But if you have a Sum Z, made by the Addition of three Numbers D, E, F, or more, then the Proof is after this manner; begin from the left hand, and say, 2, 8, and 4 make 14, which subtracted from 16, there remains 2, which with 0 makes 20; then 9, 3, 7, makes 19, which taken from 20, there remains 1, which with 8 makes 18; 7, 5, 6, make 18, which taken from 18 there remains nothing; therefore the Addition is right, seeing the Sum Z is equal to all its parts taken together.

$$\begin{array}{r} 297 \text{ D} \\ 835 \text{ E} \\ 476 \text{ F} \\ \hline 1608 \text{ Z} \\ \hline 310 \end{array}$$

To Prove Subtraction.

Subtraction is proved by Addition; Add the remainder X to the Number Subtracted H; then if their Sum K agreeth with the Number G, from which the Subtraction is made, the Subtraction is right; but wrong if not.

$$\begin{array}{r} 234 \text{ G} \\ 52 \text{ H} \\ \hline 182 \text{ X} \\ 52 \\ \hline 234 \text{ K} \end{array}$$

Also Subtraction may be proved by Subtraction, thus; Subtract the remainder X from G, and the new remainder will be L; if this agrees with the Number H first Subtracted, the first Subtraction was right; the reason of each Proof is plain.

$$\begin{array}{r} 234 \text{ G} \\ 52 \text{ H} \\ \hline 182 \text{ X} \\ 52 \text{ L} \end{array}$$

To Prove Multiplication.

Multiplication is proved by Division; Let the Numbers multiplying one another be N, P, divide the Product Q, by the multiplier P, if the Quotient R agreeth with the Multiplicand N, the Multiplication is truly performed; if not, the contrary.

$$\begin{array}{r} 523 \text{ N} \\ 6 \text{ P} \\ \hline 3138 \text{ Q} \\ 523 \text{ R} \end{array}$$

The

The Reason is manifest from the first Definition of Multiplication and Division; if the Multiplication be performed by the Rods, there is scarce any need of a proof, that Method is so secure from error; yet if you are willing to prove it, there is no proof easier than to reiterate the Multiplication.

To prove Division.

B A
2) 24, 8(124 C

2
04
4
0.8
0.8
0

Divis.

E A
9) 24, 11(803 F

24
9
2

2409 G

2

2411 H

Division is proved by Multiplication; let the Divisor B be multiplied by the Quotient C, if that produces the Dividend A, the Division is right; as is manifest from the Definition of Division and Multiplication: if after the Division any thing remains, multiply the Divisor E by the Quotient F, and to the Product G add the remainder H. If the Sum H is equal to the Dividend A, the Division is true: But if the Division is performed by the Rods, there is scarce need of any proof: but if you have a mind to prove it, the Multiplication necessary for the proof is performed by the same Rods, putting in their places, which served for the Division.

An easy way of Division.

Besides the two Methods of Division already delivered, I will add one more, which to me seems the complearest, the particular Multiplications being so dis-

disposed in operation, that from them the Division is proved without any new Multiplication or Writing out again the particular products aforesaid for Addition; and is thus.

Let the Number 4589 be given to be divided by 34.

To the left Hand of the Dividend write the Divisor 34 making a Lunula between them, also make a Lunula to the right Hand of the Dividend for the Quotient, and draw a line; then seek as in Sect. 2 Method

$$\begin{array}{r}
 \begin{array}{l}
 \times (3 \\
 1 \times 6 (3 \\
 34) \overline{) 4589} \quad (134 \\
 \underline{34} \quad 26 \\
 103 \\
 1 \\
 \hline
 4589
 \end{array}
 \end{array}$$

1. how often 34 is contained in 45. which you will find to be 1. multiply the Divisor by 1. and write the Product under the line, and subtract it from the Figures in the Dividend over it, so there remains 11. which write over the said Dividend; so you have 118 for a new Dividual; seek how often the Divisor is contained in that, which will be 3 times, the Divisor Multiplied by 3. makes 102 which I again write under the line and subtract it from the said 118 and write the remainder, if any, over it; then you have another new Dividual 169 in which the Divisor is contained 4 times, the Product of whose Multiplication being also wrote under the line, and subtracted from the Dividual, there remains 33. and so the Division is ended.

Now the Division is proved thus; add all the Figures wrote under the Dividend together, taking in the remainders, if any happen; and if the Product be equal to the Dividend, the Division is rightly performed; the contrary if not equal.

SCHOLIUM.

I Shall not here Write the ways of proving by casting away the Nines, because they are fallacious; yet shall shew the many excellent properties of the said Number.

1. Nine Measures every Number, whose Figures taken simply makes Nine, such Numbers are, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108, 117, 126, and infinite others.

2. The Figures of every Ninth Number in a natural Series of Numbers, continued infinitely, from a Unit, being taken singly & added together make 9, those Numbers only excepted, whose Figures are all Nines, of which sort is 99, 999, &c.

3. If any Number consisting of two Figures, that 9 does not measure, be divided by 9, the remainder shall be the same with that which remains, if the Figures were taken singly and divided by 9; as if 64 were divided by 9 there remains 1; now if the Figures 6 and 4 are taken according to their simple value they make 10, which being divided by 9, leaves also 1; this is manifest from 1 and 2.

4. But if the Figures composing the given Number, being taken singly make less than 9, that which they make shall be the remainder which is left, if the given Number be divided by 9: Let 33 be given these divided by 9, there remains 6, that is, so much as the single Figures 3 and 3 make.

5. Wherefore from these it follows, that of every Number divided by 9 there remains the same, which remains when the Figures of that Number are taken singly and divided by 9.

I have not read any who have opened the Reason of these things, but I will shew the Fountain in a word; Nine is less than Ten by Unity; If you rightly weigh and apply this, it will not be difficult to deduce the Properties afore-mentioned.

Moreover, although this Property is both excellent and most true, yet the Proofs made from it are fallacious; for if from the Figures of all the Numbers to be added,

simply

simply taken, 9 be cast away as often as it can; that is, are divided by 9, and the remainder be reserved; also if from the Figures of the Sum taken simply, 9 be cast away as often as it can; and if this remainder is the same with the first remainder, the Addition is concluded to be rightly performed: But the Conclusion is false; for although for the most part it does not err, yet there may be Infinite cases given, in which this Proof shall declare the Addition right when it is wrong; the reason is because 9 (or any other Number) by dividing two unequal Numbers, may give one and the same remainder; which is manifest of its self.

Example.

In the Numbers A, B, C, 3 and 5 are 8, and 8 are 16, casting away 9, and there remains 7, which with 2 makes 9, which cast away 3 and 4 make 7, therefore the Numbers A and B, being divided by 9, there remains 7: Now in the Number C, 8 and 3 make 11, casting away 9, there remains 2, which with 5 make 7; therefore in each part, both in the Numbers A B to be added, and in the pretended sum C, there remains 7; yet C is not the true Sum of A B, for they make 592, In like manner both in Subtraction, Multiplication, Division, and in the extraction of Roots the truth of this manner of proof is to be suspected.

$$\begin{array}{r} 358 \text{ A} \\ 234 \text{ B} \\ \hline 835 \text{ C} \end{array}$$

THE
SECOND BOOK
OF
PRACTICAL
Arithmetick.

Of Fractions, or broken Numbers.

CHAP. I.

*The Definitions of Fractions, and how to
Write and Express them.*

A Broken Number, which is also called a Fraction and Piece, is a Part, or Parts of a Unit, representing something whole and divisible; as if any whole were cut into five equal parts, and any should take three of them, those three fifth parts are called a broken Number.

Wherefore seeing a broken Number is the part, or parts of any whole, it must be wrote with two Num-

Numbers, whereof one may tell how many parts are taken out of the whole, and the other what parts; also those Numbers are to be wrote one over another, drawing a Line between them, as you see done in these Examples.

$$A \quad \frac{1}{2} \quad B \quad \frac{2}{3} \quad C \quad \frac{4}{7} \quad D \quad \frac{11}{13}$$

The upper Number shews how many parts are taken out of the whole, or Integer; the lower shews into how many parts the whole is divided; and so what those parts are, which are taken out of the whole.

Therefore the upper Number (because it tells the parts taken out of the whole) is called the Numerator; but the inferior (because it shews what manner of parts are taken) is called the Nominator, or Denominator.

The Fractions, A, B, C, D, are thus expressed; A one Second, or an Half; B two Thirds; C four Sevenths; D eleven Thirteenths.

But although a Unit for the most part is taken for the whole, or Integer that is Divided, yet it may be supposed to be any other Number; and so the Fraction E (if the whole or Integer be 64 Crowns) signifies five eighth parts of 64 Crowns.

To understand the Nature of Fractions thorowly; this is carefully to be observed, that Fractions do not differ from whole Numbers in the main: The only difference is, that Fractions design things which are parts of things designed by whole Numbers; and therefore the Unities of a broken Number are relative, but the Unities of a whole Number are absolute. For Example; Let 125 Pound be given, and the Fraction $\frac{4}{20}$ a, that is four Twentys of one Pound; the whole Number 125 contains a hundred and twenty five Units, whereof each is a Pound; likewise the Fraction $\frac{4}{20}$ a, that is, four twenty parts of a Pound; contains four Units; each whereof denote one twentieth part of a Pound, that is, a Shilling, and

$\frac{1}{10}$ *b*
 $\frac{3}{5}$ *c*
 $\frac{2}{5}$ *d*

and thence four twenties of a Pound signifie four Shillings, which is a whole Number, as well as 125 Pounds. Further, every whole Number designing the part of any thing is a Fraction, if it be compared with the greater whole Number, for 2 Shillings are the one tenth of a Pound, or *b*, and 3 Feet are the 3 fifths of a Pace or *c*, and 2000 Paces are the two fifths of one Italian Mile, or *d*. It concerns Learners to consider these things very well; for the chief reason why the treating of broken Numbers useth to seem so difficult and obscure to them, is that they run to learn the Operations and Rules of Fractions, before they understand their nature.

A General Observation.

When two or more Fractions are compared between themselves, they are to be understood always parts of the same whole or Integer, unless the contrary be declared.

CHAP. II.

The Theorems of Fractions.

THEOREM I.

Every Fraction is to its Whole or Unity, as the Numerator is to the Denominator.

Demonst. This is manifest from the Definition of Fractions explained in the preceding Chapter; for the Denominator denotes the whole cut into so many equal parts as there are Units in the said Denominator: But the Numerator sheweth how many parts are taken of the whole so cut. Therefore seeing a Fraction

tion is nothing else, but the Number of Parts which the Numerator sheweth to be taken of the whole; it is manifest, that the Fraction is to its whole, as the Numerator is to the Denominator; which being rightly understood thereon depend the following *Corollaries*.

Corollary 1.

WHEN the Numerator is less than the Denominator, the Fraction is less than its Whole, or Integer.

When the Numerator is greater than the Denominator, the Fraction is greater than its Integer. So the Fraction $\frac{30}{20}$ *a*, which is thirty twentys of a Pound (that is 20 Shillings) is greater than its Integer, that is, than a Pound, or 20 Shillings.

When the Numerator is equal to the Denominator, the Fraction is equal to its Integer. So *b*, that is 20 Twentys of a Pound (that is 20 Shillings) make a Pound; so *c*, *d*, are equal to a Unite, or to the whole, which a Unit doth represent.

Corollary 2.

FROM this Theorem is gathered the Truth of that which is delivered in the first way of Division, Chap. 9. Sect. 8. viz. That when a lesser Number (2) is to be divided by a greater (3) the Quotient is had; if the greater be wrote under the lesser, so as to make the Fraction $\frac{2}{3}$ *f*, for by the Definition of Division, Chap. 9. the Quotient is to Unity, as the Dividend, which here is 2, is to the Divisor, which here is 3; but also the Fraction *f* is to Unity, as 2 to 3, therefore *f* is the Quotient.

M

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THEOREM 2.

E **F** **I**F as the Numerator a of one Fraction is to its Denominator b , so the Numerator c of another Fraction, is to its Denominator d ; those Fractions are equal. Or thus, If $a \dots b :: c \dots d$; I say $E = F$.

Demonstration. By Theor. 1. $E \dots 1 :: a \dots b$; and by Hypothesis, $a \dots b :: c \dots d$; wherefore $E \dots 1 :: c \dots d$; again by Theorem 1, $F \dots 1 :: c \dots d$; wherefore $E \dots 1 :: F \dots 1$; Therefore seeing E and F have one and the same reason to Unity, by Eucl. 5. 9. $E = F$; that is, the Fractions are equal to one another. W. W. Demonstrated.

Wherefore the value of Fractions is not to be estimated by the greatness of the Numbers by which they are expressed, but by their Proportion, which shall be further explained in Theor. 3, 4, 5.

THEOREM 3.

A **B** **T**HE Fraction A , is greater than another Fraction B , whose Numerator c , hath greater reason to its Denominator f , than the Numerator g of the other, to its Denominator h .

Demonstration. For by Theor. 1. $A \dots 1 :: c \dots f$; again, $B \dots 1 :: g \dots h$; but by Hypothesis, $c \dots f$ is greater than $g \dots h$; hence also $A \dots 1$ is greater than $B \dots 1$; therefore by Eucl. 5. 10. A is greater than B ; that is, the Fraction A is greater than the Fraction B . W. W. D.

THEO.

THEOREM 4.

Those Fractions are equal, whose Numerators being multiplied into their alterne Denominators, produce the same Number.

Let the Fractions E, F be given;

if $a \times d = k$, and also $c \times b = k$.

I say the Fractions E and F are equal.

$$\begin{array}{r} \text{E} \quad \text{F} \\ a \quad \frac{2}{b} \quad \frac{3}{6} \quad c \\ \quad \quad \quad d \end{array}$$

Demonstr. By Hyp. $a \times d = k$, and $c \times b = k$, that is, $a \times d = c \times b$, therefore by Eucl. 7. 19. $a :: b :: c :: d$;

wherefore by Theorem 2. $E = F$. W. W. Demonstrated.

THEOREM 5.

THAT Fraction A is greater than another B, whose Numerator multiplying the others Denominator, produces a Number greater than that produced by its Numerator, being multiplied in the Denominator of the first.

Let $g \times n = d$, and $m \times h = c$, because d is greater than c, I say that A is also greater than B.

$$\begin{array}{r} d \quad 18 \quad 12 \quad c \\ g \quad \frac{2}{h} \quad \frac{3}{9} \quad m \\ \quad \quad \quad A \quad B \end{array}$$

Demonstr. Because by Hyp. d is greater than c; therefore by Cor. 2. Prop. 19: Book 7. Eucl. g hath greater reason to h, than m hath to n:

therefore by Theor. 3. of this Chap. the Fraction A is greater than the Fraction B. W. W. Demonstrated.

Corollary.

From the 4 and 5 Theorems is had an easie way to try whether Fractions are equal or unequal, and which is the greatest.

THEOREM 6.

FRactions (D, G) having the same Denominator (p) have the same Proportion one to the other, as their Numerators (m, n) have; that is $D \dots G :: m \dots n$.

Demonstr. By Theorem 1. $D \dots 1 :: m \dots p$; also $G \dots 1 :: n \dots p$; therefore by *Euclid* 5. 22. by equality of Reason, $D \dots G :: m \dots n$; which was to be Demonstrated.

$$\begin{array}{r} D \quad G \\ m \quad 7 \quad 5 \quad n \\ p \quad 8 \quad 8 \quad p \end{array}$$

THEOREM 7.

LET the Fractions P and R be given, and let the Numerator a of the first multiplying the Denominator o, of the other, make ao, and the Numerator c of the other multiplying the Denominator of the first n, produce cn; then the Fraction P shall be to the Fraction R, as ao is to cn; that is $P \dots R :: ao \dots cn$.

Demonstr. By Theor. 1. $P \dots 1 :: a \dots n$, and by *Hyp.* $n \times o = no$, and $a \times o = ao$; therefore $a \dots ao :: a \dots n$; wherefore $P \dots 1 :: ao \dots no$; again because by *Hyp.* $n \times c = nc$, and $n \times o = no$, therefore $b \dots no :: o \dots n$, and by Theor. 1. $R \dots 1 :: c \dots o$; hence therefore by equality of Reason $c \dots P \dots R \dots 1 :: cn \dots no$; $R :: ao \dots nc$. W. W. Demonstrated.

a Schol. 10.

Prop. 17.7.

the same.

22. 5.

$$\begin{array}{r} P \quad a \quad c \quad R \\ n \quad o \end{array}$$

j the whole.

THEOREM 8.

IF two Fractions (C, D) have the same Numerator (a) the first C, and last D shall be in reciprocal Proportion with their Denominators (o, n) that is $C \dots D :: o \dots n$.

Dr.

Demonstr. Make $axo = ao$, and
 $axn = an$, than $ao :: an :: o :: n$,
 by the Preceeding Theor. C. . D ::
 $ao :: an$; therefore C. . D :: o . n.
 W. W. Demonstrated.

$$\frac{ao}{an} = \frac{a}{o} \cdot \frac{a}{n} D$$

CHAP. III.

Reduction of Fractions.

PROBLEM I.

To Reduce a Fraction to its least Terms.

A Fraction is said to be reduced to its least Terms, when another is found equal thereto, but expressed in less Numbers.

Let the Fraction A be given to be reduced to its least Terms.

Construction. By Prop. 35. *Euc.* 7. find the two least Numbers that have the same Ratio to one another, as the Numerator n , hath to the Denominator p , which let be r and t ; then $n :: p :: r :: t$. I say the Fraction B, made of r and t , is that which is required.

Demonstr. By Construction $n :: p :: r :: t$; therefore by Theor. 2. of the foregoing Chapter, the Fraction A is equal to B; neither can there be given another Fraction in lesser Numbers equal to the said A; for, as is manifest from Theorem 2. to perform this, there are required two other Numbers in the same Proportion, as n to p , and less than r , t , already found, which is impossible to find.

But if the Terms n, p , of the given Fraction A , are the least in their Proportion, it cannot be expressed in lesser Terms; and whether the Terms n, p , are the least in their Proportion, is known by the very Construction, from *Euclid* 35. 7; and universally Numbers which are prime one to another are the least of all Numbers that have the same Proportion with them, by *Eucl.* 7. 23.

PROBLEM 2.

To Reduce Fractions to a common Denominator.

PART I.

LET there be any two Fractions N and P given, to be reduced to a common Denominator.

Constr. Multiply the Denominators together, and the Product shall be the common Denominator, as $b \times d = bd$; then multiply the Numerators into their contrary Denominators, and the Products shall be new Numerators, to the new or common Denominators, as $a \times d = ad$, and $c \times b = cb$; I say then that the Fraction $R = N$, and $T = P$.

Demon. Because by *Constr.* $a \times d = ad$, and $b \times d = bd$, hence by *Euclid* 7. 17. $ad \dots bd :: a \dots b$; therefore by *Theorem* 2. of the preceding Chap. $N = R$ in like manner, seeing $c \times b = cb$ and $b \times d = bd$, it follows that $cb \dots bd :: c \dots d$; and therefore $P = T$, wherefore the Fractions N, P , are reduced to a common Denominator, which was to be done.

PART

PART 2.

LET there be given three Fractions G, H, I, or more, to be reduced to a common Denominator.

Constr. Multiply all the Denominators b, d, f, one into the other, as $b \times d \times f$, and the Product bdf shall be the common Denominator. And multiply the Numerator of each given Fraction into all the other Fractions Denominators, and the several Products shall be the Numerators to the common Denominator; so $a \times d \times f = adf$, $c \times b \times f = cbf$, and $e \times b \times d = ebd$, I say

G	H	I
$\frac{a3}{b4}$	$\frac{c5}{d6}$	$\frac{e1}{f8}$
R	T	V
$\frac{adf}{bdf}$	$\frac{cbf}{bdf}$	$\frac{ebd}{bdf}$
R	T	V
$\frac{144}{192}$	$\frac{160}{192}$	$\frac{24}{192}$

that the Fraction $R = G$, $T = H$, $V = I$:

Demonst. By *Euc.* 7. 17. $adf \dots bdf :: a \dots b$ therefore by *Theor.* 2. $R = G$ again, because $cbf \dots bdf :: c \dots d$; therefore $T = H$. Again because $ebd \dots bdf :: e \dots f$; therefore $V = I$, and so the Fractions G, H, I, are reduced to the Fractions R, T, V, having one common Denominator, which was to be done.

The Letters shew all the Operation of each part at first sight, which excellent Property belongeth only to Specious Arithmetick.

PART 3.

LET there be any Number of Fractions, as A, B, C given to be reduced to a common Denominator, and in their least terms.

Con-

$$\begin{array}{r} A \quad B \quad C \\ \frac{3}{4} \quad \frac{5}{6} \quad \frac{1}{8} \\ \hline \end{array}$$

24

$$\begin{array}{r} 6 \quad 4 \quad 3 \\ \frac{18}{24} \quad \frac{20}{24} \quad \frac{3}{24} \\ \hline D \quad E \quad F \end{array}$$

Construction. By 36 and 38 of *Eucl.* 7. find the least Number 24, which the given Denominators 4, 6, 8, will measure, and that shall be the common Denominator; divide this common Denominator by the Denominators 4, 6, 8, and by their Quotients 6, 4, 3, multiply the given Numerators 3, 5, 1, which shall produce 18, 20, 3, for the new Numerators.

Now I say, that the Fractions D, E, F, having one common Denominator, are equal to the Fractions A, B, C, given, and also are in their least Terms.

Demonstration 1. That they are equal, from the *Construct.* and *Eucl.* 9. 19. it's manifest, $4 \dots 24 :: 3 \dots 18$; that is, by Permutation $4 \dots 3 :: 24 \dots 18$, and Inversly $3 \dots 4 :: 18 \dots 24$; wherefore by *Theor.* 2, $A = D$; by the same way of arguing, $B = E$, and $C = F$.

Demonstr. 2. That they are in their least Terms; to get Fractions of the same Denominator, equal to the given Fractions A B C; of necessity the given Denominators 4, 6, 8, must evenly divide the common Denominator sought, giving such Quotients, which if multiplied into the Numerators given, may produce the new Numerators sought, but by *Construction* the common Denominator 24, already found, is the least which the given Denominators 4, 6, 8, do evenly divide: Therefore there can be no other Fractions found, having a common Denominator, and in their least Terms equal to the given Fractions A, B, C, other than D, E, F, which are already found.

PROB.

PROBLEM 3.

To Reduce a Fraction to a Denominator assigned.

LET the Fraction F be given to be reduced to another Fraction, whose Denominator is d .

Construction. By *Euc.* 9. 19. to b, a, d find a fourth Proportional, F G
which let be c , which with the Denominator d , makes the Fraction $\frac{c}{d}$ equal to F , the Fraction given, as is manifest from *Theor.* 2.

But if b do not measure or evenly divide c , as in this second Example, where b dividing c gives the Quotient n, m , write the Integral Part of the Quotient over the Denominator d , then the Fraction $n, d, + m$ of p (that is six Eighths, with two Fifths of one Eighth) is equal to the Fraction a, b .

Demonstr. Because $b, a :: d$

$.. n, m$; it is manifest that the Number n, m , declares how many parts of d , that is how many eighth Parts must be taken out of the whole, so as to have a Fraction, whose Denominator is d , equal to the Fraction a, b ; seeing therefore every Unit of the whole Number n shews that so many eighth Parts are to be taken out of the whole; it is manifest that in the Numerator n, m , something is less than a Unit; to wit, that m sheweth somewhat else, less than one Eighth, must be taken out of the whole, viz. m, p , that is two fifths of one Eighth, which is a Fraction of a Fraction; but more of these in Chap. 8.

$$\begin{array}{r} \text{m } p \\ 24 \quad 8d \quad 6n \quad 2, 1 \\ \hline b5 \quad 8d \quad 5, 8 \\ 32^c \quad \left(\frac{n}{6} \right) m \end{array}$$

The

$$\begin{array}{r}
 20 \\
 \hline
 8 \overline{) 140} \text{ (17 } \frac{4}{5} \text{ or } \frac{34}{5}) \\
 \underline{8} \\
 60 \\
 \underline{56} \\
 4
 \end{array}$$

and the Quotient gives $17 \frac{4}{5}$, which are the Shillings equal to the Fraction given.

After the same manner may be found the value of any Fraction in the parts of the whole, best known and most common; that is by multiplying the Numerator of the Fraction given, by any parts assigned of the whole, and dividing the Product by the Denominator, for the Quotient gives the Number of Parts denominated, as was required.

Examples.

$$\begin{array}{r}
 8 \\
 20 \\
 \hline
 11 \overline{) 160} \text{ (14} \\
 \underline{11} \\
 50 \\
 \underline{44} \\
 6 \\
 \underline{12} \\
 11 \overline{) 72} \text{ (6} \\
 \underline{66} \\
 6
 \end{array}$$

How many Shillings are contained in $\frac{8}{5}$ of a Pound?

8 Multiplied by 20, produces 160, which divided by 11, quotes 14 Shillings, and $\frac{6}{11}$ of One Shilling.

How many Pence are contained in $\frac{6}{11}$ of a Shilling,

Multiply 6 by 12, the Pence in one Shilling; and the Product 72 divide by 11, and the Quotient is 6 Pence, and $\frac{6}{11}$ of a Penny.

How

How many Pounds are contained in $\frac{3}{4}$ of a Hundred?

Multiply 30 by 112, the Pounds in a Hundred Weight; the Product 3360 divide by 98, and the Quotient gives 34 Pounds, and $\frac{3}{4}$ of a Pound, which by the same Method may be Reduced to Ounces.

$$\begin{array}{r}
 112 \\
 30 \\
 \hline
 98 \overline{) 3360} \quad (34 \\
 \underline{294} \\
 420 \\
 \underline{392} \\
 28
 \end{array}$$

PROBLEM 4.

To Reduce a Fraction greater than its VVhole, (or an Improper Fraction) to a whole Number.

LET the Improper Fraction be A ; divide the Numerator b by the Denominator c , and the Quotient d , shall be equal to the Fraction A .

Demonstr. By Theorem 1. Unity being put for a Whole, $A = \frac{b}{c}$; and also by the Definition of Division, $d = \frac{b}{c}$; therefore by Euclid 5. $A = d$ which was to be Demonstrated.

But if when the Numerator is divided by the Denominator, the Quotient is a whole Number, with a Fraction, as it happens in the Fraction B , then the given Fraction cannot be reduced wholly to a whole Number; and the whole Number and Fraction taken together, shall be equal to the given Fraction; the Demonstration hereof, is the same as before.

$$\begin{array}{r}
 A \\
 b \ 12 \\
 c \ 4 \quad (3 \\
 \hline
 B \\
 c \ 14 \\
 d \ 4 \quad (3
 \end{array}$$

Upon

Upon the same reason is grounded what was added at the end of Division, whereby Pence are reduced to Shillings, Shillings to Pounds, Feet to Paces, Paces to Miles, &c. As if I would reduce 12000 Pence to Shillings; I divide 12000 by 12, and it produces 1000 Shillings, which divided by 20, produces 500 Shillings.

PROBLEM 5.

To reduce a whole Number to a Fraction, whose Denominator is given.

LET the whole Number a , be given to be reduced to a Fraction, whose Denominator may be b .

Multiply the given Denominator b , by the whole Number given a , and under your Product c , write the given Denominator b , thence is made the Fraction E ; I say $E = a$.

Demonstr. by the Definition of Multiplication, Chap. 7. $c \dots b :: a \dots 1$; also by Theorem 1. $c \dots b :: E \dots 1$; wherefore Euclid 5. 9. $a = E$. W. W. D.

If a Unit be wrote under any whole Number, as 8, it is made a Fraction, or rather something like a Fraction Equivalent to the Integer 8.

Upon the same reason is grounded what was added at the end of Multiplication, whereby Pounds are reduced to Shillings, Shillings to Pence, Miles to Paces, Paces to Feet, &c.

CHAP. IV.

Addition of Fractions.

I.

To add a Fraction to a Fraction.

IF the Fractions (as A, B) have one common Denominator, then add the Numerators 2, 6, together, and write their Sum 8 over the common Denominator 5; and the Fraction C, which arises from thence, is the Sum of the Fractions A and B given,

$$A \frac{2}{5} \quad \frac{6}{5} \quad B$$

$$C \frac{8}{5}$$

$$D \frac{2}{5} \quad \frac{3}{7} \quad E$$

$$F \frac{14}{35} \quad \frac{15}{35} \quad G$$

$$H \frac{29}{35}$$

If the Fractions have not one common Denominator, as D, E, by *Prob. 2. Chap. 3.* Reduce them to the same Denominator as F, G; the Sum of which H, is equal to D, E, given.

II.

To add an Integer to a Fraction.

BY *Problem 3. Chap. 3.* Reduce the Integer A, to the Fraction C, having the same Denominator with the given Fraction B; then add, as in *Sect. 1.*

$$A \quad 3 \quad \frac{4}{5} \quad B$$

$$C \frac{15}{5} \quad \frac{4}{5}$$

$$D \frac{19}{5}$$

III.

III.

To add many Integers to many Fractions.

ADD the Integers separately into one Summ, and add the Fractions separately into one Summ, as in Section 1. then add these two Summs together as in Section 2.

$$\begin{array}{r}
 \frac{1}{2} \\
 3 \frac{1}{2} \quad 9 \quad 9 \\
 \frac{2}{6} \quad 18 \quad 3 \quad 5 \\
 \frac{4}{9} \quad 2 \quad 8 \\
 \hline
 12 \quad 22 \\
 \quad 18 \\
 216 \quad 22 \quad 216 \\
 \quad 18 \quad 18 \quad 238 \\
 \quad \quad 18
 \end{array}$$

Let $3\frac{1}{2}$, $4\frac{2}{3}$, $5\frac{4}{9}$ be given to be added together.

The Summ of the Integers is 12; then to find the Summ of the Fractions in their least terms; I first get the least common Denominator, by *Eucl.* 38. 7, which is 18, then dividing that by the several Denominators I get the Quotients 9, 3, 2, which multiplied by the several Numerators of the Fractions given, produce Numerators to the Denominator found, making Fractions in their least terms and having a common Denominator, equal to those given: then the Numerators 9, 5,

8, added together make 22, under which writing the Denominator 18, it makes the Fraction $\frac{22}{18}$ the Summ of the Fractions given: so I had the Integer 12 and Fraction $\frac{22}{18}$ to be added together, which is done by Section 2. Thus multiply 12 by 18, and under its Product write 18, then add the Numerators together, and under their Summ write the Common Denominator; so the Fraction $\frac{238}{18}$ is the Summ of the whole Number and Fractions given which was required.

The Demonstration of all this, is sufficiently manifest in it self.

C H A P. V.

Subtraction of Fractions.

To Subtract a Fraction from a Fraction.

IF the Fractions have the same Denominator (as I and K) Subtract the lesser Numerator (2) from the greater (3) and under the remainder 1, write the common Denominator. The Fraction L thence made will be the remainder of the lesser Fraction given I, taken from the greater K.

If they have several Denominators (as M, N) reduce them to others O, P, having a common Denominator, by Prob. 2. Chap. 3. which Subtract one from the other, as before, and the remainder will be Q.

$$\begin{array}{rcl}
 & 2 & 3 \\
 \text{I} & \frac{2}{5} & \frac{3}{5} \text{ K} \\
 & 5 & 5 \\
 & \frac{1}{5} & \\
 & \text{L} &
 \end{array}$$

$$\begin{array}{rcl}
 & 2 & 3 \\
 \text{M} & \frac{2}{3} & \frac{3}{5} \text{ N} \\
 & 10 & 9 \\
 \text{O} & \frac{10}{15} & \frac{9}{15} \text{ P} \\
 & 15 & 15 \\
 & \frac{1}{15} & \\
 & \text{Q} &
 \end{array}$$

II.

To Subtract a Fraction (less than a Unit, or its whole) from a whole Number.

$$\begin{array}{r}
 S \\
 R \ 25 \ \frac{15}{17} \ a \ \frac{17}{17} \ b \\
 \quad \quad \quad \frac{2}{17} \ c \\
 \quad \quad \quad 24 \ \frac{17}{17} \ b \\
 \quad \quad \quad T
 \end{array}$$

Take a Unit from the Integer R, or a Fraction b ; b equal to a Unit, from which Subtract the given Fraction S, which is done by Subtracting its Numerator a from the Numerator b , and writing the remainder c over the common Denominator b , for the

Fraction T, arising thence with the whole Number less a Unit, is the remainder sought:

Demonst. From *Corol. 1. Theor. 1. Chap. 1.* it is manifest that the Fraction $b \ b$ is $=$ to a Unite; wherefore when I Subtract S from $b \ b$, I Subtract S from a Unite; but it is manifest from Section 1. that S is taken from $b \ b$; if I Subtract a from b , therefore S is Subtracted from a Unite, when the Numerator a is taken from the Denominator b ; the rest is clear.

III.

To Subtract a Fraction (greater than a Unit) from a whole Number, or to Subtract a whole Number from such a Fraction.

$$\begin{array}{r}
 X \ 12 \ \frac{18}{4} \ V \\
 Y \ \frac{48}{4} \ \frac{18}{4} \\
 \quad \quad \quad 2 \ \frac{30}{4}
 \end{array}$$

BY *Prob. 5. Chap. 3.* Reduce the Integer X to a Fraction Y, having the same Denominator with the given Fraction V, then Subtract the given Fraction V, from the whole Number so reduced, that is from Y, as in *Sect. 1.*

After

To Subtract a whole Number and a Fraction from a whole Number.

By Sect. 2. Chap. 4. Collect the whole Number with the Fraction (A) into one Summ (as C) which Subtract from the given whole Number (B) as in the preceding Section.

$$\begin{array}{r} 2 \\ 16 \text{ B} \\ - 6 \frac{3}{20} \text{ A} \\ \hline 10 \frac{17}{20} \text{ C} \end{array}$$

To Subtract a whole Number from a whole Number and a Fraction.

If the Integer D, which is to be Subtracted be less than the Integer E to which the Fraction F belongs; subtract the Integers one from the other, and the remainder G with the Fraction given F, shall be the remainder sought.

$$\begin{array}{r} 4 \\ 13 \text{ D} \\ - 8 \text{ E} \\ \hline 5 \text{ G} \end{array} \quad \begin{array}{r} 4 \\ 5 \text{ F} \\ \hline 5 \frac{4}{5} \text{ A} \end{array}$$

But if the Integer H, which is to be Subtracted, be greater than the Integer I, to which the Fraction K belongs, by Sect. 2. Chap. 4. Collect the Integer I with its Fraction K into one Summ (as L) and Reduce the Integer H to the Fraction M of the same Denomination with L, by Prob. 5. Chap. 3. then the Subtraction of M from L is performed as in Sect. 1.

$$\begin{array}{r} 9 \\ 10 \text{ H} \\ - 12 \text{ I} \\ \hline 2 \text{ L} \end{array} \quad \begin{array}{r} 29 \\ 2 \text{ K} \\ \hline 29 \frac{2}{2} \text{ L} \end{array}$$

N

Or

1110

$$\begin{array}{r}
 \text{I} \quad \text{K} \\
 \text{H} \quad 12 \quad 10 \frac{9}{2} \\
 \text{P} \\
 2 \quad \frac{1}{2} \quad 14 \frac{1}{2} \\
 \text{ON}
 \end{array}$$

Or thus, seeing H is to be Subtracted from I with K; H will be less than I with K; but seeing H is given greater than I; it must of necessity follow that the Fraction exceeds the excess of H above I, and thence is greater than a Unit, wherefore by Prob. 4. Chap. 5. Seek what whole Number there is in the Fraction K, and add it to I, So that O with N may be equal to I with K; now, because H must necessarily be less than O, take H from O, and the remainder P, shall be together with the Fraction N, is the remainder sought.

VI.

To Subtract a whole Number with a Fraction, from a whole Number with a Fraction.

$$\begin{array}{r}
 \text{A} \quad 5 \frac{2}{3} \quad \text{B} \quad 8 \frac{3}{4} \quad \text{E} \\
 \text{C} \quad \frac{17}{3} \quad \text{F} \quad \frac{35}{4}
 \end{array}$$

LET the Integer A and its Fraction B, which is to be subtracted, be collected into one Summ C, by Sect. 2. Chap. 4. In like manner get the Summ F, of the other two (D, E.) Then reduce these Sums to the same or a common Denominator, by Prob. 2. Chap. 3. and perform the Subtraction as in Sect. 1.

The Reason of all these Operations is plain from themselves

CHAP.

CHAP. VI.

Multiplication of Fractions.

I.

To Multiply a Fraction by a Fraction.

TO Multiply A by B, Multiply the Numerators *om* one into another, and their Product is *om*; also Multiply the Denominators *p, n*, one into another, and their product is *pn*, then *om* shall be the Numerator, and *pn*, the Denominator of the Fraction C, produced by the Multiplication of the Fractions A, B.

$\begin{array}{r} A \\ \frac{0\ 2}{p\ 3} \\ C \\ \frac{8}{15} \end{array}$	$\begin{array}{r} B \\ \frac{4\ m}{5\ n} \\ C \\ \frac{om}{pn} \end{array}$
--	---

II.

To Multiply a Fraction by a whole Number.

TO Multiply D by E, Multiply the whole Number E by the Numerator *k* of the Fraction D, and under the product *n*, write *m*, the Denominator of the given Fraction D, then the Fraction F thence arising shall be the Product sought.

$\begin{array}{r} D \\ \frac{k\ 2}{m\ 3} \end{array}$	$\begin{array}{r} E \\ 5 \\ 1 \end{array}$
$\begin{array}{r} F \\ \frac{n\ 10}{m\ 3} \end{array}$	

N 2

Q1

Or thus, suppose a Unit to be wrote under the whole Number E, and the Multiplication will be as in Sect. 1.

III.

To Multiply a whole Number, and a Fraction, by a whole Number.

LET the whole Number with its Fraction be collected into one Summ, by Sect. 2. Chap. 4. then the Multiplication is performed, as in the preceding Section.

Or Multiply the whole Number by the whole Number separately; and then the Fraction by the whole Number.

IV.

To Multiply a whole Number, and a Fraction, by a whole Number and a Fraction.

BY Sect. 2. Chap. 4. get the Summ of one of the whole Numbers and its Fraction, and the Summ of the other whole number, and its Fraction; and multiply these two Summs together, as in Sect. 1.

Demonstration.

THE Operation of the 1 Section is only to be demonstrated, for the Demonstration of the rest will follow from that; it is required therefore to shew that $B \cdot C :: 1 \cdot A$, that is, that B multiplied by A produces C, as is manifest from the Defn. of Multiplication, Chap. 7.

$m \times p n = p n m$, also $n \times o m = o m n$, therefore by Theorem 7. Chap. 2. $B \dots C :: p n m \dots o m n$, and by Eucl. 7. 17. $p m n \dots o m n :: p \dots o$; but by Theo. 1. Chap. 2. $1 \dots A :: p \dots o$, that is $B \dots C :: 1 \dots A$ W W D.

$$\begin{array}{r} p n m, \quad o m n, \\ o \quad m \quad o m \\ \hline p \quad n \quad p n \\ A, \quad B, \quad C, \end{array}$$

SCHOLIUM.

WHEN the Fractions to be multiplied together are each less than a Unit, the Product is less than either, which seems strange to them that are not well vers'd in Numbers; but the reason hereof is soon demonstrated from the Definition of Multiplication.

For Example, if $A \times B = C$, then $1 \dots A :: B \dots C$, therefore if 1 be greater than A, also B shall be greater than C, but if $B \times A = C$; then again $1 \dots B :: A \dots C$, and because 1 is greater than B, therefore also A shall be greater than C.

$$\frac{2}{3} A \quad \frac{3}{4} B \quad \frac{6}{12} C$$

If one of the Fractions to be Multiplied be greater than a Unit, then the Product shall be less than that, but greater than the other.

N 3 CHAP.

CHAP. VII.

Division of Fractions.

I.

To Divide a Fraction by a Fraction.

$$\begin{array}{r} \text{A} \quad \text{B} \quad \text{C} \\ a \frac{6}{8} \text{ by } \frac{m \ 2}{n \ 4} \left(\frac{o \ 3}{p \ 2} \right. \end{array}$$

$$\begin{array}{r} \text{A} \quad \text{B} \quad \text{E} \\ a \ 4 \quad m \ 3 \quad n \ 5 \\ \hline \text{c} \ 9 \quad n \ 5 \quad m \ 3 \end{array}$$

$$\text{D} \left(\frac{a \ n \ 20}{c \ m \ 27} \right)$$

D Ivide A by B: If the Terms m, n, of the Divisor B, measure the Terms a and c, of the Dividend A; the Numbers o and p, by which they are measured, constitute the Fraction C, and is the Quotient sought.

But if the Terms of the Divisor B, do not measure the Terms of the Dividend

A, Invert the Divisor B, and it will be E; then by it so Inverted, multiply the Dividend A, and the Product D shall be the Quotient sought.

Or thus: If the Terms of B, do not measure those of A; multiply the Denominator of the Divisor, by the Numerator of the Dividend, and its Product shall be a new Numerator a n; also the Numerator of the Divisor, multiplied by the Denominator of the Dividend, produces c m, which is a new Denominator, and so the new Fraction D is the Quotient sought.

II.

To divide a Fraction by a whole Number, or a whole Number by a Fraction.

$\frac{2}{3}$ by $\frac{6}{1}$ $\frac{2}{18}$ Suppose a Unit to be placed under the whole Number, and the Division is done, as in Sect. 1.

III.

To divide a whole Number with a Fraction, by a whole Number with a Fraction.

LET each be collected into one Sum, by Sect. 2 Chap. 4. then between the two Sums the Division is performed, as in Sect. 1.

IV.

To divide a large whole Number with a Fraction, by a whole Number.

AS for Example: Let 598, with the Fraction A, be given to be divided by 3, you may work this conveniently, thus; divide the Integer 598 by 3, and it quotes 199. if any thing remains, as here 1, add that to the Fraction A, by Sect. 2. Chap. 4. whence you have the Sum B, which divide by 3, as in Sect. 2. and write the Quotient C, to the Integral part of the Quotient, then 199 with C, will be the Quotient sought.

$$\begin{array}{r} 3 \overline{) 598 \frac{A}{3}} (199 \frac{B}{3} \\ \underline{597} \\ 1 \end{array}$$

Demonstration.

THE first Operation in *Self*. 1. only needs Demonstration, for the rest will be manifest from this. In the first place let m , and n , measure a , and c , by o , p ; because by Supposition m measures a , by o ; therefore by *Euclid* 7. Axiom 8. $m \times o = a$; again, because n measureth c , by p , therefore $n \times p = c$; therefore by the Demonstration of the preceeding Chap. $B \times C = A$; therefore by the Definition of Multiplication, $1.. B::C..A$; wherefore by Permutation, $1..C::B..A$; therefore by the Definition of Division, Chap. 9. Book 1. C is the Quotient of A , Divided by B .

But when the Terms of the Divisor B , do not measure the Terms of the Dividend A , then it seemeth very strange that Division should be performed by Multiplication.

$$F = \frac{mna}{mnc} \quad m n$$

$$A \quad B \quad E$$

$$\frac{a}{c} \text{ by } \frac{m}{n}$$

$$D = \left(\frac{an}{cm} \right)$$

Wherefore it remaineth to Demonstrate, that if A be multiplied by the Divisor B Inverted, that is by E , the Product D shall be the Quotient of A , Divided by B .

The Terms of the Divisor multiplied one into another, produce mn ; that is, $m \times n = mn$, and $mn \times a = mna$, and $mn \times c = mnc$, of which the Fraction F is made; by *Euclid* 7. 17. $mna..mnc::a..c$; therefore by Theorem 2. Chap. 2. $F \pm A$: But the Numerator m of the Divisor B , dividing mna , gives the Quotient na (for as Specious Multiplication is performed only by setting letters together, so division is by taking away of letters) and the Denominator n , of the same Divisor B , dividing mnc gives the Quotient cm ; therefore by the first part hereof, the Fraction B divideth the Fraction F , by the Quotient D a Fraction; wherefore

since

since I shewed before that $F = A$, also B divided A by the Quotient D, the same that arises from the Multiplication of A, by the Divisor B Inverted; that is by E, which was to be Demonstrated.

SCHOLIUM.

Here also it seems strange to raw Arithmeticians, that in Division of Fractions the Quotient is found for the most part greater than the Number which is divided; which always happens, when the Divisor is less than a Unit; but the reason hereof is manifest from the Definition of Division; A divided by B, gives the Quotient C; therefore by the Definition of Division, $1 \dots C :: B \dots A$; that is by Permutation, $1 \dots B :: C \dots A$, but by Supposition 1 is greater than B, therefore also C is greater than A.

$$\begin{array}{ccc} A & B & C \\ \frac{5}{4} & \text{by } \frac{2}{3} & \left(\frac{15}{8} \right) \end{array}$$

CHAP. VIII.

Of Fractions of Fractions.

AS from whole Numbers cut into Parts, arise broken Numbers or Fractions, so from the parts of whole Numbers sub-divided in other lesser parts, are had Fractions of Fractions; as if I were to take A out of B; that is 3 fourths of 16 Twentys of one Pound, they are wrote as in the Examples here set; A, B, are 3 fourths of 16 Twentys; C, D, are

$$\begin{array}{ccc} A & B & \\ \frac{3}{4} & \text{of } \frac{16}{20} & \\ C & D & \\ \frac{2}{5} & \text{of } \frac{1}{6} & \\ E & F & G \\ \frac{2}{5} & \text{of } \frac{1}{6} & \text{of } \frac{3}{7} \\ & & 2 \text{ fifths} \end{array}$$

2 fifths of one Sixth; E, F, G, are 2 fifths of one Sixth, of three Sevenths.

For Compendiousness, a Fraction of a Fraction may be called a second Fraction; a Fraction of a Fraction of a Fraction, may be called a third Fraction, &c.

Before the Arithmetical Operations of Fractions of Fractions can be Instituted they must first be reduced to simple Fractions.

To Reduce a Second Fraction to a Simple Fraction.

$$\begin{array}{r} \text{B} \quad \text{D} \\ \frac{2 \ a}{3 \ c} \text{ of } \frac{p \ 3}{n \ 4} \\ \hline \text{Z} \\ \hline \frac{ap \ 6}{cn \ 12} \\ \text{E} \end{array}$$

LET the second Fraction B, D, be given to be reduced to a simple Fraction; multiply the Numerators a and p, one into another, which makes a p; and also the Denominators c, n, one into another, which make c n, and the simple Fraction E, made of them, shall be equal to the second Fra-

tion B, D.

Demonstration. Find Z by this Analogy, $p \dots n :: c \dots Z$; it is manifest that D is the Whole of the second Fraction B, D, and consequently that B, D $\dots D :: a \dots c$; but by Theorem 1. Chap. 2. the simple Fraction D $\dots 1 :: p \dots n$; that is by Construction $c \dots Z$; wherefore by equality of reason B, D $\dots 1 :: a \dots Z$, again, because by Construction $p \dots n :: c \dots Z$; therefore by *Euclid* 7. 19. $pZ = cn$.

Wherefore seeing $p \times a = ap$, and $p \times Z = cn$, as is shewn before; therefore by *Euclid* 7. 17. $a \dots Z :: ap \dots cn$; but before it was shewn that B, D $\dots 1 :: a \dots Z$; therefore B, D $\dots 1 :: ap \dots cn$; and by Theorem 1. Chap. 3. $ap \dots cn :: E \dots 1$; therefore by *Euclid* 5. 11. B, D $\dots 1 :: E \dots 1$; therefore by *Euclid*

Euclid 5. 9. the second Fraction B, D, and the simple Fraction D, are equal, which was Proposed.

To reduce a third Fraction, or any other to a simple Fraction.

LET the third Fraction C, D, E, be given to be reduced to a simple Fraction.

Reduce the second Fraction D, E, to I a simple Fraction, then C, is the Fraction of the Fraction I; again, reduce the second Fraction C, I, to the simple Fraction G, and it shall be equal to the third Fraction C, D, E given, as is manifest from the Construction itself.

$$\begin{array}{r} \text{C} \quad \text{D} \quad \text{E} \\ \frac{3 \text{ a}}{5 \text{ b}} \quad \frac{4 \text{ n of } 5}{6 \text{ p z y}} \\ \text{G} \quad \frac{\text{any}}{b \text{ p z}} \quad \frac{ny}{p z} \end{array}$$

The whole Operation is performed after this manner; all the Numerators are multiplied one into another, and also all the Denominators, that is $a \times n \times y = a n y$, and $b \times p \times z = b p z$, and the Fraction G, thence arising, will be that required.

Questions to Exercise the foregoing Rules of Fractions:

Quest. 1. **W**HAT Number is that, from which if $3 \frac{1}{2}$ be subtracted, the remainder will be $4 \frac{1}{3}$?

Answer. $7 \frac{1}{6}$.

2. What Number is that which being added to $3 \frac{1}{3}$, the Sum will be $7 \frac{1}{2}$?

Answer. $4 \frac{1}{3}$.

3. How much is the Fraction $\frac{2}{3}$, greater than the Fraction $\frac{1}{4}$?

Answer. $\frac{1}{12}$.

4. What

4. What Number is that, which being divided by $4\frac{1}{2}$, quores $2\frac{1}{8}$? *Answer, $8\frac{4}{8}$.*

5. What part of $2\frac{1}{2}$ is $\frac{3}{4}$? *Answer, $1\frac{1}{2}$.*

6. What Number is that which being multiplied by $3\frac{1}{2}$, produces $12\frac{1}{2}$? *Answer, $3\frac{1}{2}$.*

7. How many Eighteenths, or $\frac{1}{18}$, are there contained in $\frac{1}{9}$? *Answer, $\frac{1}{9}$.*

This is performed by Division, for if $\frac{1}{9}$ be Divided by $\frac{1}{18}$, as in *Se^{ct}. 1. Chap. 3.* the Quotient will be $\frac{1}{9}$ required.

8. How many thirds are there contained in $3\frac{1}{2}$?

Answer, $10\frac{1}{2}$.

9. How many Eighths, or $\frac{1}{8}$, must be added to $\frac{3}{4}$ to make $\frac{8}{9}$? *Answer, $2\frac{1}{4}$.*

To resolve this, first find to what Number $\frac{3}{4}$ must be added to make $\frac{8}{9}$, which is done by subtracting $\frac{3}{4}$ from $\frac{8}{9}$, and you have $\frac{1}{4}$; then seek how many $\frac{1}{8}$ are contained in $\frac{1}{4}$, which is found by dividing $\frac{1}{4}$ by $\frac{1}{8}$, and you have $2\frac{1}{4}$; therefore if $\frac{3}{4}$, and $2\frac{1}{4}$ of $\frac{1}{8}$'s be added to $\frac{3}{4}$, the Sum will be $\frac{8}{9}$, as was required.

10. From how many Sevenths, or $\frac{1}{7}$, must $\frac{3}{4}$ be subtracted, so as that $\frac{1}{2}$ may remain? *Answer, 7 Sevenths, and $\frac{7}{10}$ of $\frac{1}{7}$.*

The resolution of this is like the foregoing, using Subtraction instead of Addition.

11. What Number is that, from which if the difference between $\frac{3}{4}$, and $\frac{1}{5}$, be Subtracted, the remainder will be the difference between $\frac{1}{2}$ and $\frac{1}{9}$?

Answer, $\frac{3}{4}$.

To resolve this get the difference between $\frac{3}{4}$ and $\frac{1}{5}$, which will be $\frac{11}{20}$; also get the difference between $\frac{1}{2}$ and $\frac{1}{9}$, which will be $\frac{7}{18}$, then add these two differences together, and their Sum is $\frac{31}{36}$, from which Subtract the first difference, to wit, $\frac{11}{20}$, and the remainder shall be the other difference, to wit, $\frac{1}{9}$.

12. By how many Fifths, or $\frac{1}{5}$, must $2\frac{1}{2}$ be multiplied to produce $7\frac{2}{3}$? *Answer.* $15\frac{1}{3}$

To resolve this seek a Number which multiplied by $2\frac{1}{2}$, may produce $7\frac{2}{3}$, which will be $3\frac{1}{15}$; then seek how many Fifths are contained in $3\frac{1}{15}$, which you will find to be $15\frac{1}{3}$.

13. What Number is that by which if $7\frac{1}{2}$ is divided, the Quotient will be $\frac{1}{3}$ of $17\frac{1}{2}$? *Answer.* $\frac{1}{3}$

14. By how many 9's must $7\frac{1}{2}$ be divided to quote $2\frac{1}{4}$? *Answer.* 30

CHAP. IX.

Decimal Fractions.

THE Arithmetick of broken Numbers, commonly called Vulgar Fractions, whose Theory and Praxis we have already explained, although necessary and pleasant to know, yet what great trouble there is in large Calculations, and many Repetitions, every one knows who are conversant in Numbers; but as the Labour in Division almost wholly vanishes by *Napiers Bones*; so we are freed from the troublesome of Vulgar Fractions, by the Excellent Invention of *Decimals*; the Praxes of which are performed in an excellent Compendium, as if they were whole Numbers.

What

What Decimal Fractions are, and how to Write them.

Decimal Fractions or Numbers, are the Tenth, Hundredth, Thousandth, &c. Parts of any Whole, that is are such Fractions whose Denominators proceed in a decuple Progression from Unity, as 1, 10, 100, 1000, &c. which are wrote after the Vulgar manner like *a*, which is three tenths, and *b*, which is seven hundredths; but because the Denominators of these Fractions

have no other Figures but a Unit and Cyphers, the Calculation will be rendred more expeditious; if the Denominators are not exprest under the Numerators, but over them, by certain Signs or Indices, as is done in Astronomical Minutes; and thence may be called *Primes, Seconds, Thirds, Fourths, &c.* according as the Index denotes, and according to their places reckoned from the left hand, as you see they are exprest hereunder.

Primes, Seconds, Thirds, Fourths, Fifths.				
Index. I	II	III	IV	V
Value. 10	100	1000	10000	100000 &c.
Tens, Hundreds, Thouſ. Tens Thouſ. Hun. Thou.				

Therefore the value of the Decimal *a, b, c, d, e, f*, is this; *a* is three Tenths or Primes; *b* four Hundredths, or Seconds, *c* six Thousandths, or Thirds; *d* two Ten Thousands, or Fourths; *e* nine Hundred Thousands, or Fifths; *f* four Millions, or Sixths: But if to the Figures which are thus marked, or supposed to be

be marked belong others which are not signed as *gh*, then they are to be estimated as whole Numbers, from hence it is manifest that the order of places

i	ii
8	3 2 1
g	h k l

in any Decimal Fraction proceedeth from the Left hand towards the Right, contrary to the Order of places in Integers, which is from the Right hand towards the Left.

When a Decimal Fraction is to be expressed, two things must be wrote, to wit, the Number, and the Index; the Number shews how many Decimal parts must be taken out of the whole, and the Index sheweth what parts.

The end of Decimal Fractions is to perform all Arithmetical Operations, appertaining or belonging to the use of Man, without vulgar Fractions.

CHAP. X.

Requisites needful to the Demonstration of Decimal Arithmetick.

IF we proceed Methodically, the Demonstration of Decimal Operations, will be made very easie.

Definitions.

1. **T**Hose Figures which have Indices over them, must not be valued according to their places, as whole Numbers, but must be estimated as single Figures, (that is, as if every one stood in the

i	ii	iii
2	3	4
a,	b,	c

the first place) hence if the Decimal Figures a, b, c , are given, although a stands in the Third place, it doth not signify 200 Tenths, but 2 Tenths;

Likewise, although 3 standeth in the Second Place, it doth not signify 30 hundredths, but 3 hundredths, &c.

2. But if any whole Number be joined to the Decimal Fraction, it must be estimated of the same value, which it would have been estimated at, if the Decimals were wanting; so the Figures $p q$, although they precede the Decimal Figures a, b , are valued at 48 not 4800.

3. When the Decimal Fraction is wrote like a vulgar Fraction, the Denominator hath a Unit, and so many Cyphers after it as are denoted by the Index, so the Decimal Fraction e wrote after the vulgar way is the Fraction,

ii	6 n
6	—
e	100 m

on, n, m , whose Denominator m , hath two Cyphers, that is so many as are designed by the Index (11.)

iii
K 3
1000

4. The Value of a Decimal Index is a Unit with so many Cyphers as is denoted by that Index, as if the Decimal Number K were given, the value

of the Index is 1000. this is manifest from Definition 3.

Axioms

IF to any number B, whether it be wholly an Integer, or an Integer and a Decimal Fraction together, there be added Cyphers having decimal indices over them, they shall not change the value of the said Number.

	B	C
	1	ii iii
23	0	0 0
1	11	iii iv
57	0	0 0
B		C

For the Cyphers have no value of themselves, neither being placed before the Integer B, do they raise its place, or increase its value, as is manifest from Definition 2. This Axiom will be illustrated hereafter.

In like manner, when the Decimal Rank A is interrupted, if it be continued by interposing Cyphers, the value of the given Decimal A, will not be changed.

	1 iv
A. 4.	3 5
1	ii iii iv
4.	3. 0 0 5

THEOREM I.

THE Decimal Fraction, a, b, c, d, is equal to the Vulgar Fraction O, whose Numerator is the given Figures a, b, c, d, (without the Indices) estimated according to their local Value as Integers, and Denominator the value of the greatest Index; that is, a Unit, with so many Cyphers as are denoted by the greatest Index iv.

Q

De

Demonstration. Write the Decimal Parts, *d*, *c*, *b*, *a*, each at large; that is, make the Fractions

$$\begin{array}{ccccccc} \text{I} & \text{II} & \text{III} & \text{IV} & & & \\ 7 & 2 & 3 & 4 & \text{---} & \text{E} & \text{---} & \text{I} \\ a & b & c & d & 10000 & & 10000 \end{array}$$

$$\begin{array}{ccccccc} 7 & 2 & 3 & 4 & & & \\ \text{---} & \text{O} & \text{---} & \text{F} & \text{---} & \text{K} & \\ 10000 & & 1000 & & & & 10000 \end{array}$$

$$\begin{array}{ccccccc} & & & & 2 & & 200 \\ & & & & \text{---} & \text{G} & \text{---} & \text{L} \\ & & & & 100 & & 10000 \end{array}$$

$$\begin{array}{ccccccc} & & & & 7 & & 7000 \\ & & & & \text{---} & \text{H} & \text{---} & \text{M} \\ & & & & 10 & & 10000 \end{array}$$

$$\begin{array}{r} 4 \\ 30 \\ 200 \\ 7000 \\ \text{---} \\ 7234 \end{array}$$

E, *F*, *G*, *H*, whose Numerators are the said Figures simply taken, but Denominators the values of their Indices. Also make other Fractions, *I*, *K*, *L*, *M*, whose Numerators may be the same Figures, but estimated according to their local value as Integers, and the value of the greatest Index their common Denominator.

I will now shew that the Fractions *I*, *K*, *L*, *M*, are equal to the Fractions *E*, *F*, *G*, *H*. Let us compare the two Fractions *L* and *G* together: because in whole Numbers the value of places proceeds

in a decuple Progression, from the right hand to the left; but in Decimals, the value of Signs or Decimal places proceeds in a decuple Progression from the left hand towards the right; it is manifest that the value of the Figure *b*, estimated according to its Integral place, (that is, the Numerator 200 of the Fraction *L*) exceeds the simple value of the said Figure *b*, (that is, the Numerator of the Fraction *G*) as the value of the greatest Sign (*IV*) (that is, the Nominator of the Fraction *L*) exceeds the value of the Sign (*II*) with which the Figure *b* is marked (that is, the Denominator of the Fraction *G*) hence $2 \cdot 100 :: 200 :: 10000$; therefore by Theor. 2. Ch. 2. the Fractions *L* and *G* are equal; by like reason

reasoning $I = E$, $K = F$, $M = H$. But the Fractions I , K , L , M , constitute the Fraction O , because (10000) the value of the greatest Index (iv) is their common Denominator, and their Numerators being collected together, make 7234, as is manifest in its self; therefore also the Fractions E , F , G , H ; that is, the Decimal given d, c, b, a , is equal to the Fraction O . W. W. Demonstrated.

THEOREM 2.

If a whole Number (m, n) hath annexed to it a Decimal Fraction (a, b, c) the whole Number with the Decimal shall be equal to the Fraction P , whose Numerator is all the Figures given (m, n, a, b, c) omitting the Indices where they are, and reckoning the Figures according to the value of their places; and the Denominator (1000) the value of the greater Sign (iii).

Demonstration. By Theor. 1. the Decimal a, b, c , is equal to the Fraction d, e , and if the whole Number m, n , be multiplied by e , and the product f be wrote over e , so as to make the Fraction f, g ; the same shall be equal to the Integer m, n , by Prob. 5. Chap. 2.

But the Fractions f, g , and d, e , make the Fraction P . For seeing the Nominator is common to both Fractions, the Numerators f, d , are only to be added together, whose Sum must of necessity consist of the Figures given, m, n, a, b, c , as is evident by a small consideration thereof; therefore also the whole Number m, n , with the Decimal Parts a, b, c , make the Fraction P . W. W. Demonstrated.

	i	ii	iii
\$ 2.	5	4	9
m n	a	b	c
f 32000		549	d
g 1000		1000	e
549			
32000		32549	h
32549		1000	e
			P

THEOREM 3.

i iv ii v
 1 2 3 0 4 5275
 i ii iii iv ii iii iv
 c, 20 0 3, c 5207 0 0 5
 2003 e 5207003 e
 —————
 10000 d 100000 d

Fraction a.

The Demonstration hereof, is the same as of Theorem 1 and 2.

Corollary.

FROM these Theorems the Axiom before Theo. 1. may be further explained: Take d , and to it add three Cyphers, noted with Indices, so as to make f ; by Theorem 1. $d = gh$; and $f = km$, by the same Theorem; but by how many Cyphers k exceeds g , by so many Cyphers m must of necessity exceed h ; therefore by Theorem 1. Chap. 3.

Book 2. $k..g::m..h$, and by permutation, $k..m::g..h$; therefore by The. 2. Chap. 2. Book 2. the Fraction $km = gh$; wherefore seeing $km = f$, and $gh = d$; therefore also, $d = f$.

This indeed is manifest in its self, and thence the Axiom which I proposed; yet it seemed necessary because of the great use of this Axiom, to write this Declaration thereof.

CHAP. XI.

Addition of Decimals.

LET A, B, C, be given to be added together; if the Progression of the Indices are either interrupted, as in C, or are wanting, or break off, as in B, continue the Progression, by interposing Cyphers, as in E, or adding Cyphers, as in E; (by which supplement the value of B 861. C or B are not changed) then write like Indices, under like Indices, as is here done in D, E, F, and add them together as if they were whole Numbers, and over the first Figures of the Sum G, write the same Indices or Signs, as in the Numbers given to be added, and so G, with the Indices, shall be the Sum required.

$$\begin{array}{r}
 \text{A } 35.2471 \text{ D. } 35.2471 \\
 \text{B } 861. \\
 \text{C } 861. \\
 \text{E } 861.0000 \\
 \text{F } 7.0409 \\
 \text{G } 903.2880 \\
 \text{I } 352471 \\
 \text{K } 8610000 \\
 \text{L } 70409 \\
 \text{N } 9032880
 \end{array}$$

Demonstration. By the Theorems Demonstrated in the preceeding Chapter, D, E, F, are equal to the Fractions I, K, L, whose Numerators are D, E, F, taken as Integers, and the value of the greatest Index their common Denominator. Now to get the

the Sum of these Fractions, we need only add the Numerators together, that is, D, E, F, as if they were whole Numbers, and write the common Denominator p, under the Sum, as is manifest from Sect. 1. Chap. 4. Therefore, also to get the Sum of the given Decimals, D, E, F, we only need to add them together as Integers, and under their Sum to write p, the value of the greatest Index, whereby is had the Fraction N; but the Fraction N, by the Theorems of the preceeding Chapter, is equal to the Sum G, for the Numerator thereof hath the same Figures with G, and the Denominator p, is the value of the greatest Index (iv) by Hypothesis; therefore G is the Sum of the given Decimals D, E, F, or A, B, C, which was to be Demonstrated.

APPENDIX.

AS the Writing of Decimal Fractions with Indices or Signs over them, does not want its use and excellency, they best expressing the value of each Figure; yet for common Operation, the writing them is both tedious and perplexing; therefore later Arithmeticians omit the Indices; and reckon the value of the Decimal parts mentally, from their Places or Denominators, setting a Point to the left hand of the Decimal, to distinguish it from a whole Number; which Method we will also joyn to this of using the Indices, in all the Operations of Decimals.

So the Numbers A B C, or D E F, are expressed without Indices, by *d, e, f*; wherein observe, that like degrees or places are wrote under one another; that is, Primes, under Primes, Seconds under Seconds, and Thirds under Thirds, &c. which must always be minded,

d 44. 2471
e 861. 0000
f. . 7. 0409
g 903. 2880
h

ed. And then the Addition is the same as in whole Numbers, only after the Addition is ended, due regard must be had to the placing of the Punctum, to distinguish the whole Numbers from the Decimals; to do which remember this generally, that there must be as many Decimal parts in the Sum, as are in any of the Numbers given to be added; So in this Example, they having each four Decimal Figures, the Point must be placed after the fourth figure in the Sum that is after 2; and so the Sum hath the whole Number 903, and Decimal Fraction 2880.

CHAP. XII.

Subtraction of Decimals.

LET B be given to be Subtracted from A. Write the lesser whole Number under the greater, as usual, and like Decimals one under another, supplying the vacant places in the Indices, both in the beginning, as in A, and in the middle, as in B, with Cyphers signed, as is done in C and D; by which suppletion of the given Decimals A and B, the value is not changed by the Axiom in Chap. 2. then Subtract D from C, as if both were Integral or

11		11111V
A 98.42	C 98.	42000
1111V		11111V
B 4594	D .4.	05904
		11111V
	Q 94.	36096
E 9842000		F 405904
100000		100000
	T 9436096 q	
	100000 p	

O 4

ab.

absolute Numbers, and mark the first Figures of the remainder Q , with the same Indices as the other Numbers C or D have, and this shall be the remainder sought.

Demonstration. By the Theorems of Chap. 10. C , D , are equal to the Fractions E , F , whose Numerators are C , D , taken as Integers, and common Denominator p , the value of the greatest Index: Now to Subtract F from E , we need only Subtract the Numerator from the Numerator, that is D taken as an Integer, from C taken as an Integer, and write the common Denominator, that is, the value of the greatest Sign p , under the remainder.

Therefore to Subtract D from C , that is by the Axiom, A from B , we need only to take D as an Integer, from C as an Integer, and under the remainder Q to write p , the value of the greatest Index, whence will be had the Fraction T for the remainder. But the Fraction T , by Theor. 2, is equal to Q , for by Hypothesis, the Numerator q , hath the same Figures as Q hath, and the Denominator p is the value of the greatest Sign; wherefore Q is the difference of C and D , or the remainder required. *W. W. Demonstrated.*

A P P E N D I X.

d 98. 42000

c 4. 0904

e 94. 36096

ing the same Cautions as are given in Addition.

THE Numbers above, or any other Numbers having Decimals, may be Subtracted one from the other, without writing the Indices, observing

CHAP.

CHAP. XIII.

Multiplication of Decimals.

LET A and B be given to be Multiplied together. The Multiplication of A and B must be performed (without any regard to the Indices) like whole Numbers; (first taking away the interruption of the

Decimal Progression, if there be any, by interposing Cyphers, as in the second A) then add together the greatest Indices of the given Numbers A and B, and their Sum shall give (iv) the greatest Sign by which the first Figure of the Product C must be marked; which Index will also

1 III	G	H
7 4 A	704	52
11111		
A 7 0 4	1000 e	10 p
1		
B 5 2	36608 k	
14 0 8	10000 f	
35 2 0	N	
11111111		
366 0 8 C		

show how many Figures are to be marked with Indices decreasing orderly.

Demonstration. By the Theorems of Chap. 10. A and B are equal to the Fractions G and H, whose Numerators are A, B, taken as whole Numbers, and Denominators the values of the greater Signs; but to multiply the Fractions G and H one into another, we need only multiply their Numerators, that is, A and B taken as whole Numbers, one into another, producing a new Numerator k (to wit, C without the Indices) and under the same to write the Denominator

nator f , consisting of a Unit, and the Cyphers e, p , taken together; that is the value of both the greatest Signs of A and B together, therefore also to multiply the Decimals A and B together, we only need multiply A and B one into another, as Integers, producing the Numerator k , and under the same write the Nominator f , that is, the value of both the greatest Signs of A and B added together, so getting the Fraction N for the Product: But by Theorem 2. Chap. 10. $N = C$, for the Numerator k , hath the same Figures that are in C, and the Denominator f , consists of a Unit, and the Cyphers e, p , that is by Hypothesis, of so many Cyphers as are denoted by both the greatest Signs of A, B together; that is, by Construction, of so many Cyphers as are denoted by the greatest Sign in C; therefore also C is the Product sought by the Multiplication of the given Decimals A, and B, one into the other. W. W. D.

If one of the given Numbers be an Integer, having no Decimals belonging to it; the first Figure of the Product must be signed with the greatest Index of the other Number given, and the rest in order, with Indices decreasing.

APPENDIX

Multiplication is better performed for common use without Indices, and then the general directions for Multiplying are these.

Multiply the Numbers given, as if they were whole Numbers, and cut off always from the Product by a Point, so many places towards the right hand, as there are places of Decimal parts in both the Numbers given to be Multiplied; that done, the Figure or Figures (if any happen to be) on the left hand of the said Point, are whole Numbers, and those

those on the right Hand are Decimal Parts.

Let 4.38 be given to be Multiplied by 2.65, that is, a whole Number and a Decimal Fraction, by a whole Number and a Decimal Fraction:

Write the Numbers given to be Multiplied one under another, and multiply them like Integers, so getting the Product 116070; then because there are four decimal places in the Multiplicand and Multiplier, cut off four Figures to the right hand from the said Product, so it will stand thus, 11.6070; hence the true Product is 11.6070, or 11.6070.

$$\begin{array}{r} 4.38 \\ 2.65 \\ \hline 2190 \\ 2628 \\ 876 \\ \hline 11.6070 \end{array}$$

Let .384 be given to be Multiplied by .269; that is, a Decimal Fraction, by a Decimal Fraction; having got the Product 64896, then because the Multiplicand and Multiplier, have together 6 Decimal parts or places, cut off six Figures to the right hand, from the said Product, but here happening to be but five, I must add a Cypher to the left Hand, and after that place the Point; so the Product will stand thus, .064896, and are all decimal Parts.

$$\begin{array}{r} .384 \\ .269 \\ \hline 3456 \\ 2304 \\ 384 \\ \hline .064896 \end{array}$$

Let .704 be given to be Multiplied by 5.2; that is, a Decimal Fraction, by a whole Number and a Decimal Fraction; having got the Product 36608, because the Multiplicand and Multiplier contain together four places, cut off four Figures to the right hand, from the said Product, so it will stand thus; 3.6608.

$$\begin{array}{r} .704 \\ 5.2 \\ \hline 1408 \\ 3520 \\ \hline 3.6608 \end{array}$$

The Demonstration of what is here added, is the same as if the Indices were used, for the Number of places in the Multiplier and Multiplicand together are the same with the Sum of their greatest Indices,

as is apparent from the Declaration of Decimals; in Chap. 9. and from what is added in Addition of Decimals, Chap. 11.

Multiplication of Decimals Continued.

BECAUSE it oftentimes falls out in the Multiplication of Decimal parts, that there is no need to express all the Figures of the Product, but only some of them towards the left hand, those towards the right being but of small value, and in many Operations not any way necessary; therefore we will exhibit a Method to Multiply decimal Fractions together so, that the Product shall have no more parts or places then desired.

P R A X I S.

WRite the Units place of the Multiplier under that Figure of the Multiplicand, whose Index is equal to the Number of Decimal parts designed to be left in the Product; then revert the remaining Figures of the Multiplier, and Multiply every Figure of the Multiplier into so many Figures of the Multiplicand, as stand equal with it towards the left hand, having always respect unto what would have been brought, if the next Figure to the right hand in the Multiplicand had been Multiplied, and set the first Figure of every particular Product under the first Multiplying Figure, and the other Figures in order,

Ex.

Example,

LET 5.6839 be given to be Multiplied by 4.329 ; but so that the Product may have but three Decimal parts.

Having set 4, which is the Units place in the Multiplier, under the third Decimal part of the Multiplisand, because the Product is to have 3 Decimal parts, and reverted the Multiplier, as you see done here: Multiply 5683 by 4, thus; Say 4 by 9 is 36, for which I carry 4 in my Mind; 4 times 3 is 12, and 4 added makes 16; I write down 6 under 4, and proceed as is usual.

$$\begin{array}{r}
 5.6839 \\
 9\ 234 \\
 \hline
 22736 \\
 1705 \\
 114 \\
 60 \\
 \hline
 24.615
 \end{array}$$

Again, to Multiply by 3, say 3 times 3 is 9, for which keeping 1 in my Mind, 3 times 8 is 24, to which 1 being added, it makes 25; then writing 5 under 4, that is under the first multiplying Figure; proceed as is common.

Again, 2 times 8 is 16, for which reserve 2, and 3 times 6 is 12, to which the 2 reserved being added, it makes 14, write down 4 under 5, and proceed.

Lastly, 9 by 6 is 54, for which reserve 6, and 9 in 6 is 54, to which adding 6 reserved, the Sum is 60, the first Figure of which is wrote under 4.

Then the particular Products being added together their Sum is 24615, from which must be cut off three Figures to the right hand for the Decimal parts designed, as you see done in the Scheme; so the Product will be 24.615, consisting of an Integer and but of 3 Decimal Parts, as was required.

Let it be required to multiply 145.684 by 135.38, so as that the Product may have only Integers.

Having

Having set 6, the Units place of the Multiplier under 5, the Units place of the Multiplicand, the first Figure to the right Hand of the Product being to be in the place of Units, revert all the other Figures of the Multiplier which when reverted will stand thus 84631, and then the Work is performed, as in the Example foregoing, and as is done in the annexed Scheme, producing 19881, without any Decimal Fraction, as was required.

Let 4898 be given to be Multiplied by .3468, and so as that the Product may have two decimal parts, because to the Multiplier there belongs no Integer, therefore set a Cypher in the place of Units; and because in the Multiplicand there are no decimal parts, annex Cyphers thereto, and then the Multiplication will be as in the first Example; see the Scheme, from which all is plain:

Demonst. To Demonstrate this Construction, set the first Example multiplied at large, and also contracted.

Why the Units place of the Multiplier 4 is set under that Figure of the Multiplicand 3, whose Index is equal to the Number of places to be in the Product is because Units place having no Index, its Index alone is equal to the greatest Index of the Product.

Al

Also why the first Figure of the Multiplication of 568 by 3, which is 5, must stand under Units, which is the third place of decimal parts, is, because the Indices of 8 and 3, make also three; and the like reason serves for the rest.

Why the Multiplier is reverted, is because the greatest particular Product should stand first, and the rest in order under it; but that it is the same as if not reverted, is manifest, by comparing the two Schemes together.

And why every particular Figure in the Multiplier, multiplies first not that over it, but the next to the right hand, and carries the tens mentally to be added to the product of the multiplying Figure and that over it, is to make the contraction want as little as may be, of being equal to the Product of the Multiplication at large; which is also manifest, by comparing the two annexed Schemes.

$$\begin{array}{r}
 5.6839 \\
 4.329 \\
 \hline
 51 \quad | \quad 1551 \\
 113 \quad | \quad 678 \\
 3705 \quad | \quad 17 \\
 22735 \quad | \quad 6 \\
 \hline
 24.605 \quad | \quad 6031 \\
 111111 \\
 5.6839 \\
 111111 \\
 9234 \\
 \hline
 22736 \\
 1709 \\
 114 \\
 50 \\
 \hline
 24.605
 \end{array}$$

C H A P. XIV.

Division of Decimals.

$$\begin{array}{r}
 \text{11v} \\
 \text{A. } 25879 \\
 \text{111111v} \quad \text{111} \\
 \text{A } 2580079 \quad \text{B } 573 \\
 \text{111v} \quad \text{111111} \\
 \text{X } 433 \quad (4502 \text{ C} \\
 \hline
 \text{2580079p} \quad 573q \\
 \text{D } \underline{\hspace{1cm}} \quad \text{E } \underline{\hspace{1cm}} \\
 \text{100000f} \quad 100g \\
 \hline
 \text{433} \quad 4502h \\
 \text{Z } \underline{\hspace{1cm}} \quad \text{N } \underline{\hspace{1cm}} \\
 \text{100000} \quad 1000k \\
 \hline
 \text{1} \quad \text{111} \\
 \text{b } 23 \quad m56 \\
 \text{111111v} \quad \text{111} \\
 \text{k } 230000n(410
 \end{array}$$

LET it be required to divide A by B. If the decimal Progression be interrupted, fill the vacancies by interposing Cyphers (as in the second A) and divide A by B, as if they were both whole Numbers; then if the greatest sign of the divisor B, be less than the greatest sign of the dividend A, subtract that from this, and with the remaining Index (iii) mark the first Figure of the Quotient C, and mark the rest in Order, with Indices decreasing.

But if, as in this Example, the greatest Index of the Divisor (*m*) be greater than the greatest Index of the Dividend (*b*) add so many Cyphers to the Dividend (as in *k*) that from their Indices, the Divisors may be Subtracted.

The

The like is done when the Divisor (q) is absolutely greater than the Dividend (p) howsoever the Signs or Indices are.

Demonstr. By Theor. 2. Chap. 10. $A = D$, and $B = E$; But to Divide D by E , we need only Divide p by q (that is A taken as an Integer, by B as an Integer) and f by g ; which last is done by taking the Cyphers in g , from those in f ; that is the greatest Index (u) of the given Number B , from the greatest Index of the given Number A . Therefore also to divide A by B , we only divide A taken as an Integer, by B taken as an Integer; and under its quotient h , write a Unit, with those Cyphers which remain when the Cyphers of g , are taken from them of f ; so getting the Fraction N for the Quotient: But by Theor. 2. Chap. 10, $N = C$. For the Numerator h consists of the same Figures as are in C , (for they are both gotten by the Division of the same Numbers) and the Denominator k (as I will presently shew) is the value of the greatest Index of C ; therefore C is the Quotient arising by the Division of A by B . W. W. D. And that k is the value of the greatest Index of the Quotient C , I thus shew; by Construction the greatest Index of C , is that which remained when the greatest sign of B , was taken from the greatest sign of A ; but g and f are the values of the greatest signs of A and B ; and so by Defin. 4. they consist of so many Cyphers as are indicated by the greatest signs, or Indices of the said A and B ; therefore it is manifest, that if the Cyphers in g are taken from the Cyphers in f , there will remain k , the value of the greatest Index of the Quotient C .

The other case wherein Cyphers are added to the Dividend h , to make k , we thus demonstrate: By the first part already demonstrated, m divideth k ,
P

Demonstration. Because the value of the remainder X is not changed by the annexing of Cyphers, Xr together shall be equal to X alone; but Xr divided by B , quotes r ; therefore also X divided by B , quotes r ; but $A - X$ divided by B , quotes C , (for A divided by B , leaves X) therefore by dividing all A , by B , the Quotient is Cr .

The Praxis may be contained in a few words, viz. annex certain Cyphers to the remainder of the decimal Division, and continue the Division, and sign the Figures then produced with Indices, orderly increasing from the last Quotient.

Corollary.

AFTER the same manner what remains by dividing a whole Number by a whole Number, may be made to vanish; the Demonstration whereof, is as above.

APPENDIX.

AS Multiplication, so Division is better performed for common use without Indices, and the general Directions for Dividing, are these.

Divide the Numbers given, as if they were whole Numbers, and cut off always from the Quotient by a Point, so many places towards the right hand, as being added to the places of decimal Parts in the Divisor, will be equal to the places of decimal Parts in the Dividend; that done, the Figure, or Figures, (if any happen to be) on the left hand of the said Point are whole Numbers, and those on the right, are decimal Parts.

Let it be required to divide 11.6070 by 4.38 that is, a whole Number and a decimal Fraction, by a whole Number and a decimal Fraction.

Write the Numbers given to be divided, as in the third way of Division, at the end of Chap. 11. Book 1. and divide as in whole Numbers, so getting the Quotient 265. Then because there are four decimal places in the dividend, and but two in the divisor, cut off so many Figures in the Quotient towards the right hand (to wit two) that being added to the decimal places in the Divisor, the Sum may be equal to those in the Dividend; so the Quotient will stand thus; 2.65.

Let it be required to Divide .0125 by .5; that is a decimal Fraction, by a decimal Fraction.

Having divided as in whole Numbers, and got the Quotient 25; then because the Quotient and Divisor together, must contain as many decimal parts as are in the dividend, which here are four; therefore to the two places in the Quotient prefix a Cypher, and after that place the point of separation, so have you three places of decimal parts in the Quotient, to wit, .025, which with one place in the Divisor, are equal to the places of decimal parts in the dividend.

Let it be required to divide .8564, by .008.

Having

Having divided as in whole Numbers, there remains at the last Division 45 and to get its value in the Quotient, add a Cypher (or Cyphers if required) to the Dividend, and continue the Division; the reason whereof is demonstrated in Sect. 3. of this Chap. then to find the value of the Quotient 10705, cut off two Figures to the right hand, which with the three Decimal places in the Divisor, are together equal to the Decimal places in the dividend, to wit, five including the Cypher annexed; so the Quotient will stand thus 107.05.

The Demonstration, of what is here added, is the same, as if the Indices were used, for the places of Decimal parts, in the Quotient equal to the difference of the places of Decimal parts, in the Dividend and Divisor is the same with the difference of their greatest Indices; and that the places of Decimal parts in the Quotient and Divisor, must be equal to those in the Dividend, is manifest from the proof of Division; for Multiplying the Divisor by the Quotient, the Product is equal to the Dividend, hence the Dividend (equal to the Product;) must contain so many places of Decimal Parts, as are in the Multiplier (equal to the Divisor) and Multiplyer (equal to the Quotient) together, therefore having the places of Decimal parts in the Dividend and in the Divisor, the Decimal parts of the Quotient are consequently known, being no other then the difference between the places of Decimal parts in the Dividend and Divisor; which was to be shewn.

P 3 Division

Division of Decimals contracted.

When in Division of Decimal Fractions, the Divisor hath many Figures, and it is required to draw out the Quotient till the remainders of the Division may not be of small value; the operation will be large and tedious, but is efficiently contracted by the Method following, giving as few or as many Decimal parts in the Divisor as is desired.

In this contraction two things are to be considered; to wit, whether the Dividend be absolutely greater than the Divisor, or the Divisor absolutely greater than the Dividend. Let A be given to be divided by B.

$$\begin{array}{r}
 \text{B) } 381 \overline{) 512349} \quad \text{C} \quad \text{---} \\
 \underline{14} \\
 172 \\
 \underline{614} \\
 108 \\
 \underline{462} \\
 627 \\
 \underline{138} \\
 14
 \end{array}$$

Having resolv'd how many places of Decimal parts are to be in the Quotient C which let be three, make the places of Decimal parts in the Dividend and Divisor, each equal thereto by adding Cyphers if they want as in B or cutting of Figures to the

right hand if they exceed as in A. So the given dividend and divisor will be E. F. this being done.

If the Dividend A be absolutely greater than the Divisor B, and if the whole Divisor be contained under ten times in the Dividend, as it is in the annexed Scheme; see how often it is contained, in the Quotient which here is 2, and therewith Multiply the Divisor, writing the Product under the Dividend and subtract it from the same; then striking off one Figure to the right hand of the Divisor by placing a point under it as under o. see how oft the remaining Figures 231 are contained in 614 the remainder

manner of the Division or new Dividual which is twice, set 2 in the Quotient, and by it multiply the said remaining Figures of the Divisor (adding to the Product the same as any product by that figure in the Quotient multiplied by the last Figure cut off, or the next to the right hand) and write the first Figure of its product under the first of the preceding Product, or of the Dividend, and the other Figures in order; then having Subtracted the Product from the last dividual, proceed and cut off another Figure in the Divisor as 1, by placing a point under it, and with the remaining Figures 23. do as before, & continue lessening the Divisor one Figure to the right hand at every Operation until all have points under them; and so the contracted Division in the annex'd Schem quotes 2267 which by placing a point before the 3 Decimals designed to be in the Quotient, will stand thus 2. 267 or thus 2. $\frac{267}{1000}$.

2. But if the Divisor C be contain'd more than ten times in the Dividend E, place so many Cyphers to the right hand in the Divisor, making E still it cannot be contained 10 times

$$\begin{array}{r}
 \begin{array}{l} 8. 24 \\ C \end{array} \overline{) \begin{array}{l} 3 \\ 28 \\ .522 \\ .2270 \\ .36230 \\ 448.230 \\ 412000 \\ 33960 \\ 1648 \\ 494 \\ 25 \end{array}} \\
 \begin{array}{l} E \\ 8. 2400 \end{array} \overline{) \begin{array}{l} 36230 \\ 448.230 \\ 412000 \\ 33960 \\ 1648 \\ 494 \\ 25 \end{array}} \quad \left(\begin{array}{l} 54.263. \end{array} \right.
 \end{array}$$

and then proceed in all respects as before, the Quotient in the annex'd Schem, being to contain 3 decimal parts.

3. If the Divisor E be absolutely greater than the

$$\begin{array}{r}
 23.0000 \quad E \quad 3 \\
 \quad \quad \quad F \quad 2280 \\
 23.00 \overline{) \begin{array}{l} 4580 \\ 2300 \\ 2070 \\ 207 \end{array}} \quad \left(\begin{array}{l} 1991 \\ .01991 \end{array} \right. \\
 \quad \quad \quad 2
 \end{array}$$

Di

Dividend F. and so is contained more than ten times in F. cut off so many Figures to the Right hand of the Divisor till the remaining Figures C are contained less then ten times in the Dividend : which done proceed in all respects as before.

Observe always, that if but one Figure be cut off in the Divisor, the first Figure in the quotient towards the left, is Primes, If two Figures be cut off then place a Cypher for the first Figure in the quotient, and the first Figure got by Division will be seconds : If three Figures be cut off, two Cyphers, and so on.

The Praxis of this contraction may be contained in a few words without considering how many Fractions are to be in the quotient and is this : having your dividend A, and divisor B given to be divided contractedly, make the Figures in the dividend and divisor equal if the first Figure to the left of the divisor be not greater than the first Figure to the

Left hand of the dividend ; but if it be greater then the divisor, must have one Figure less than the dividend, this done, proceed in the Division as before directed, so getting the quotient 1.9956, to know the value of which is the great-

$$\begin{array}{r}
 48.324 \quad 96.4342 \\
 \quad \quad 27 \\
 \quad \quad .269 \\
 \quad \quad 4.618 \\
 \quad \quad 48.110 \\
 48.324 \quad 96.434 \quad 1.9956 \\
 \quad \quad \quad 48.324 \\
 \quad \quad \quad 43.492 \\
 \quad \quad \quad 4.349 \\
 \quad \quad \quad 212 \\
 \quad \quad \quad 24
 \end{array}$$

est difficulty : Therefore to get that, observe under what Figure of the dividend, the Unites place of the Divisor would stand if wrote under it, for the first Figure in the quotient towards the left hand will be of the same value, so in this Example, the Unites place 8. of the divisor would stand under the Unites place

place of the Dividend, and therefore the value of 1 is Unites, and all the rest are Decimals; therefore between 1 and 9, place a point, and the quotient will stand thus; in its proper and true value, 1.9956.

The reason of all these Rules of contraction is plain from what is Demonstrated in contracted Multiplication, *Chap. 13* For the divisor multiplyed contractedly by the quotient, must of necessity produce the dividend, and consequently the Unites place of the quotient; must be under such a place of Decimal parts, in the divisor, whose Index is equal to the greatest Index of the dividend; and all the other Figures of the said quotient inverted.

Hence, in the First Example, because the Units place of the quotient 2, must stand under the third Decimal place of the divisor; the Cypher was added to the right hand: and because in the Second Example the Units place 4 in the quotient, must stand under the third Decimal place of the divisor, and the place of tens 5 (the quotient being inverted) will stand under the fourth Decimal place; therefore two Cyphers are added to the right hand of the divisor: And in the Third Example, because two Figures were cut off to the right hand of the divisor the first Figures of the quotient reverted must stand under the second Decimal place of the divisor; but to make the Product of the Divisor multiplyed into the quotient, the same with the dividend, which hath 4 Decimal places, the Units place of the quotient must be wrote under the Fourth Decimal place of the divisor, which done to fill up the vacancy between 1 and Unity place, you must write a Cypher, from which likewise the reason of the observation in *Self. 3.* is manifest.

CHAP. XV.

Reduction of Decimal Fractions.

IF the greatest Integer of Money, Weight and Measure, &c. were subdivided Decimally, the Doctrine of *Arithmetick* would be taught with much more ease and expedition than now it is to essay which shall add the manner of reducing Fractions, and the Subdivisions of whole Numbers to Decimal Fractions.

I.

To Reduce a Vulgar Fraction to a Decimal Fraction.

A Single Fraction may be reduced into a decimal of the same value, or infinitely near (for all Vulgar Fractions cannot be exactly reduced to Decimals) by *Prob. 3. Chap. 3. Book 2.* thus Multiply the Decimals Denominator assigned, by the Fractions Numerator, and divide the Product by its Denominator and the Quotient shall be the Numerator to the Denominator assigned.

Let it be required to reduce $\frac{5}{8}$ to a Decimal, whose Denominator is assigned to be 1000.

Multiply 1000 by 5, and divide the product 5000 by the Denominator 8 of the Fraction given and the quotient will be 400 which is the Numerator sought therefore $\frac{400}{1000}$, or $\frac{4}{10}$ is the Decimal Fraction equal in value to $\frac{5}{8}$.

Let

Let $\frac{7}{240}$ be required to be reduced to a Decimal Fraction, whose Denominator shall be 100000.

Multiply 100000 by 7, and divide the product 700000 by 240, and the quotient will be 2916 and somewhat more; yet less then $\frac{1}{100000}$ of a Unit, which is the Numerator sought, therefore $\frac{2916}{100000}$ or $\frac{2916}{100000}$ is the Decimal Fraction almost equal to $\frac{7}{240}$.

It will happen in the Reduction of most Vulgar Fractions, to Decimals, that the Decimal will want something of being equal to the Vulgar Fraction, and therefore the Denominator of the Decimal must be assigned to be so great that what is wanting in the Numerator may be an inconsiderable Value.

The Demonstration of this, is the same as *Prob. 3. Chap. 3. Book 2.* which is grounded on *Theo. 1. Chap. 2. Book 2.* where two Fractions are proved to be equal, if the Numerator of one is to its Denominator as the Numerator of the other is to its Denominator from the inverse of which, to wit, that the Denominator of one Fraction is to its Numerator, as the Denominator of another is to its Numerator, is had the Rule above.

II.

To Reduce the Known Customary parts of Money Weight, &c. to Decimal Fractions.

THE Reduction of the Subdivisions of Money, Weight and Measure, &c. to Decimals, differs not at all from that of vulgar Fractions to decimals, for if a whole be divided into parts, and some of them be taken, these are the Numerator of a Fraction whose Denominator is all the Parts; so a Pound Sterling, being divided into 20 parts or shillings if 8 of them be taken, they shall be equal to a Fraction, whose Numerator is 8, and Denominator 20.

The

The like is to be understood in every Sub-division of Money; Weight, Measure, &c.

Hence then to Reduce the known Customary parts of Money, &c. to Decimals respect must be had to the Integer Sub-divided; for that being the Denominator and the Parts taken the Numerator, it is reduced to a Decimal, whose Denominator is assigned by the same Rule as above.

Let it be required to Reduce 12 s. to the Decimal of a Pound, whose Denominator may be 10000 because the Integer a Pound is 20 s. by the Rule above 12 Multiplied into 10000, produces 120000, which being divided by 20, quotes 6000; so shall $\frac{6000}{10000}$ or $\frac{111111}{100000}$ or $\frac{1}{1000}$ (for Cyphers increase not the value of a Decimal) be the Decimal Fraction of a l. and is exactly equal to 12 s.

Let it be required to reduce 6 d. to the Decimal of a Pound, because a pound contains 240 d. therefore multiplying the Decimal Denominator 1000 by 6 and dividing the Product by 240 is had the Decimal $\frac{1000}{40}$ or $\frac{1111}{1000}$ or $\frac{1}{1000}$ of a pound equal to 6 d.

SCHOLIUM.

THe Praxis of the foregoing may be more compendiously exprest thus to Reduce a Fraction, $\frac{a}{b}$ to Decimals, annex Cyphers to the Numerator a , and being so increased divided it by the Denominator b ; and the quotient d , thence arising shall be the Decimal, equal to the Fraction $\frac{a}{b}$ given; for because by the *Axiome* $a = f$ therefore also b dividing, each of them will give equal Quotients; but b dividing a , quotes $\frac{a}{b}$, by *Cor. 2. Theor. 1 Chap. 1.* and b dividing f , quotes d , by *Hypothesis*, therefore also $\frac{a}{b} = d$.

$$\begin{array}{r} 11111 \\ 3^a \ 3000 f \\ 7^b \ 11111 \\ \hline 428 d \end{array}$$

To Reduce a Decimal Fraction to another Fraction.

It is very necessary to know, how after any operation is wrought by Decimals, the Product may be estimated in another Fraction, or by the known and Customary Divisions or parts of whole Numbers as if it were required to know what Fraction having its Denominator 20, the Decimal B is equal to, or which is the same thing how many Shillings the said Decimal is valued at, which is obtained after this manner.

Multiply the Decimal 728 given by 20, whose Product is 14560, from which take so many of the first Figures 560, as are indicated by the greatest Decimal sign, or as there are places in the given Decimal, and the remainder 14 is the Numerator sought, therefore the Decimal B makes the C parts of 20 £. or one pound, to wit, fourteen twentieths, that is 14 s.

$$\begin{array}{r} 728 \\ \times 20 \\ \hline 14560 \end{array}$$

Again to get the Value in pence of the remainder 560, which is a Decimal, having like Indices with that given by Sect. of *Multiply. Chap.* Multiply it by 12 cutting off likewise so many Figures as at first, so have you remaining 6, which are Pence, and $\frac{720}{1000}$ part of a Penny.

$$\begin{array}{r} 560 \\ \times 12 \\ \hline 6720 \end{array}$$

After the same manner may a Decimal Fraction be reduced to any other Fraction or to the known Subdivisions of Money, Measure, Weight, and time &c. the Praxis whereof shall express in a few words thus; Multiply the Decimal Fraction given by the Denominator assigned, which is the Integer divided into such parts as are desired, and from

from the right hand cut off so many Figures as there are places in the the Decimal given and what remains to the right hand, shall be the Numerator to the Denominator assigned or the parts of the Integer required.

Demonstration. The Fraction A, whose Numerator hath the same Figures as are in B, and Denominator a Unit with so many Cyphers as are assigned by the greatest Index of B; is by *Theorem 1. Chap. 10.* equal to the Decimal B. But by *Prob. 2. Chap. 3.* to reduce the Fraction A to another, whose Denominator is 20, the Numerator 728 must be multiplied by 20, and the product 14560 divided by the Denominator 1000, which is done by cutting off so many of the first Figures as there are Cyphers in 2000, that is as are denoted by the greatest upper Index of the said A; hence therefore the truth of the prescribed Operation is manifest.

The End of the Second Book.



THE

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F I N I S

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to ^{ch} add a gill of spirits of Turpentine

When y^e Sheep is struck make a
Circle round y^e Maggots with some of
the Water dropping it out of a Bottle

This will prevent their getting away
into y^e Wool. Then drop a little among

them, Rub It about with y^e Finger

which will presently kill them.



Acid's any Thing wth Effects of
Togue wth a Sense of Sharpness or Bitterness. Are all of y^e Kind
of Salts. To distinguish dubious Acids, from Alkalies, mixe
them with a blue Tincture of Violets, if they turn it red, they
are Acid; if Green Alkaline. S^r J. Newton says, That
y^e Particles of Acids are of a size greater than those of Water
and therefore less Volatile: That they are endued wth a very
great Attractive Force, & by y^e readily mixe wth Water. Water
has no great dissolving Force, because there is but a small
Quantity of Acid in it; for whatever strongly attracts, and
is strongly attracted may be reputed an Acid, and consequently
all porous Menstruums must belong to y^e Class. Ch^m. Yet
Spirit of Urine readily dissolves Iron, or Copper, even in y^e Cold,
is allowed an Alkali; and accordingly makes a vehement Efflu-
with Aqua Fortis. Acids are prescribed in Medicine, as in
Antiscorbutics &c. — A little Diatriac leaves an agreeable
Acidity in Water.

— Air — Mr. Boyle estimates y^e Weight
of Air to y^e of Water, at a Medium as, 1, to 1000. But by
Experiment made since, before y^e Royal Society it was found
to be about 870. ^{as 1, to 870.} But there is no fixing any precise Ratio
since not only y^e Air but y^e Water itself, is continually vary-
ing; made in diff^t Places, vary, in regard of y^e different height
of y^e Places, & y^e different Consistencies of Air arising therefrom.
The Weight of Air is in proportion to its perpendicular
Altitude, a Column of Mercury being found to descend at y^e
rate of 1/2 of an Inch, for every hundred foot of Ascent.
The Weight of Air upon every Square Inch is about 15 lb
— The most necessary Proxies, as pepper, Ginger, Salts, Spirit
— have a considerable Taste on y^e Top of height.

Mountains, as on y^e Pike of Teneriff, for want of their
Particles being pressed upon y^e Tongue, so as to enter its
Pores, but instead thereof being digested, & blown away
by its heat. The only thing that there retains its Savour
is Canary Wine, w^{ch} is owing to its Unctuous quality, in
virtue whereof, it adheres closely to y^e part, & is not easily
blown away.

Gold Coin - The proportion of Alloy for
Standard Gold is 2 Carats in a pound ^{Troy} w^t of Gold

Sterling Silver 18 penny w^t alloy in all Troy
W^t. Alloy used in Gold Coins is Silver & Copper, That
in Silver Coins Copper alone

^{for dyspepsia & Spectacles}
Absorbents in Medicine, Remedy & softness or porosity
of their texture, & their proper use, & the virtues of sheep
herbage, & the use of the ordry away superfluous Moisture,
such are testaceous pores, & some Wood, as Sandal, Sassafras,
Alcalies have y^e Effect of Absorbents on Acids.
Absorbents tend opposed to Laxatives. Mint, red Roses, Nettles,
Sloe, cinnamon, Allum Shalk are purgative. Astring: They act by thick-
ning the mucus, they cannot run off so fast as before, or contracting
the parts, & diminishing y^e Pores. — Utripe Sloes, bred to
y^e Consist: of acid Extract is used as a substitute for Truellaesia
It is binding, & on that account good against Fluxes

Prepared Page 111. The prescribed to cut tough
Phlegm, & dissolve coagulated Blood: yet there are some Acids
which evidently coagulate y^e Animal Fluids & produce Astriction
in all their train of Conseq. Thus Milk readily curdles, wth Spirit of
Acid. — Receipts — Detect. N^o 1. w^{ch} is
Boil together in a pint of Water
an Ounce of Quick Lime & 1/2 Oz. of Flower Brimstone
when cold bottle & cork for use all few drops put into
glass of Wine & Sydes contain. — Let, turn & burn, if

Siphon — a hydraulec Instrument
consisting of a hollow, metalline Ball, wth a slender Neck, or
Pipe arising from y^e same; wth being fill'd wth Water, & thus
exposed to y^e Fire, produces a Vehement blast of Wind.

Sometimes y^e Neck is made to screw into y^e Ball, because then
y^e Cork may more readily be fill'd wth Water — Chauvin thinks
y^e Ball might be apply'd instead of Bellows for Fire-side) Chaud

— M^r. Boyle subject to bleed at y^e Nose found
Moss of a dead Man's Skull, tho' only apply'd to touch
y^e Skin till y^e Moss was warm thereby, y^e most Effectuall
of several Medicines he had made use off.

Aphorisms or Diobstruents or opening Roots are Smallage,
Fennel, Asparagus, Parsley, Butchers Broom, Capers
promote due Circulation of Juices by opening y^e Obstructions
passages of y^e small Vessels, Glands, & Ducts.

White Lead or Ceruse is Lead coroded in Vinegar
Thin Plates of Lead are roll'd up & put in a
Place in Earthen Potts contain: Vinegar. These
Potts being cover'd to keep out Dust tho' to suffer the
Vapour of Vinegar to escape; a g^l. of Horse Dung is
known. In y^e morning y^e. Heat of y^e. raises y^e. Vinegar
Vapour, w^{ch} corodes y^e. Lead into white Scales, w^{ch} are
wth ground in a Mill is fit for Use — It is sometimes made
wth Urine instead of Vinegar. — Neither Ceruse, nor Lithar-
geum have any Taste, but boil'd in Distill'd Vinegar by
Solution or by stalliz'd give One of y^e. sweetest Substances
in Nature, call'd Sugar of Lead, This, or Letharge, tho'
poisonous, is us'd for recovering Sour Wines, It not
only corrects y^e. Acidity of some Wines, but gives a

Barometer — His phenomena various
and causes assigned by several Authors are widely different
nor is its use in predicting of Weather yet perfectly ascertained.

On y^e Top of Snowden hill, 1240 Yards high D. Waller
found y^e Mercury lower by 3 $\frac{3}{4}$ Inches, than at y^e foot
thereof; Whence it appears that at every 30 Yards y^e Mercury
sinks 10 of an Inch. M^r. Dorcham from some Experiments
he made at y^e Top & Bottom of y^e Mount. allows 32 y^d.
perpend: Ascent to a Fall of the Mercury of 10 of an Inch
whence would y^e height of y^e Atmosphere be found 5 $\frac{1}{10}$ Miles
were it equally dense every where & ^{this} an accurate method
of measuring y^e height of Mountains.

The greatest height of Mercury has been known to stand at
London, is 30 $\frac{3}{4}$ Inches, Lowest 28 $\frac{1}{4}$ Inches
Paris 28 $\frac{1}{2}$, Lowest 26 $\frac{3}{4}$ Inches of y^e
Paris foot is as y^e Lond: foot by 100

The greatest heights of y^e Mercury are on Easterly &
North Easterly Winds. — That after great Storms of Wind
when y^e Mercury has been Low, it rises again very fast
— In calm & rainy Weather it stands high — That Northern
Places find greater Alteration of y^e Southern, and that
within y^e Tropics & near them, there is little or no Variation
of y^e height of y^e Atmosphere — Instance at Naples it
hardly ever exceeds an Inch. The Effect of Cold Bathing
is attributed not only to its choline & constringe Power, but
also to y^e Weight of y^e Water — Suppose a Person Immersed 2 feet
y^e Water of his Skin 15, he sustains a W^t. of Water, added to y^e
y^e 2280 lb. Besides the Water in Bathing enters the
y^e Cold & Viscid. This as well as y^e other Effects
of y^e Bathing are attributed to y^e Weight of y^e Water.

Condens — (Infused in Brandy) —
with Iron Filings, gives a black
at y^e heat of Boiling, the Colour becomes perfectly fine & so
gives a Dye to Cloth w^{ch} no Workmⁿ can imitate

Borax a mineral urinous Salt
of y^e vitrous kind, chiefly us'd in soldering, & fusing
of Metals; sometimes also in medicine as an
Emetic & promoter of delivery — Borax unites
with acids both mineral & Vegetable, & absorbs
them & together with them forms new salts of
diff^t kinds, according to y^e species of acid employ'd

For Malt Liquor some use Castile Soap,
some Flour & Eggs & some Sal panaristius instead
of Yeast —

Camphere — Distils from
a Tree on y^e Coast of India, a white, shining
transparent friable, Inflamable, odouriferous,
soluble Gum, or Resin of a bitterish Taste & very
hot in y^e Mouth, It is exceeding Inflamable
so as to burn & preserve its Flame in Water
and in burning it consumes wholly, leaving no
residuum behind — In Medicine y^e most effica
diaphoretick known; its great subtilty diffusing
Itself through y^e Substance of y^e part, almost as
soon as y^e Warmth of y^e Stomack has set it in
motion. It is us'd in caries of y^e Bones as a
detergent in Wounds, to resist Gangrenes &c.

Camphor dissolved in Spirit of Wine is call'd Oil of Camphor
Camphor boiled in Aqua Vitæ in a close place, till y^e Water
wholly be evaporated, y^e lighted Candle be introduced, & the

Drawing Objects

Provide a Square of $\frac{1}{2}$ of Glass fitted into a Frame
Tab: Perspec (fig: 15) and wash it over wth water wherein
a little Gum has been dissolved. When it is well drin^d
turn it towards y^e Object, so as that y^e Whole may
be seen through a Sight, G: H. fixed thereto. Then
applying y^e Eye to y^e Sight, wth a Pen & Ink draw
every Thing on y^e Glass as you see It appear theron
Lastly lay a fair moist Paper thereon; & pressing it
pretty close, you'll have y^e Whole on y^e Paper.—

In y^e Year 766, being y^e last Year of
Augustus's reign, that Prince together wth
Cicero. numbered y^e Citizens of Rome & were
found 4 millions 137 thous: Persons. Claudius
made a new Computation, in y^e Year of Christ
48, when a^d Tacitus relates, the Roman
Citizens throughout y^e whole Empire amounted
to 6 millions 96 thousand; though others
represent y^e Num^b: greater — A very rare, &
Indisputable Medal of Claudius, never yet made
Publick, co^pies y^e N^o. in y^e List made by Claud
(w^h was call'd Ostensia) to be 7 millions fitt to
bear Arms, besides all y^e Soldier on foot
Arms, & amounted to 50 Legions & Cohorts
& 60 Soldiers —

its Life 5 hours)
Ephemeron — of y^e winged or
Fly kind; & appears usually ab^t St Johns Tide —
It is born about 6 Clock Evening & dies about 11^o
But It lives 3 Days under that of a Worm in Clay Cell
or Case before It assumes this Figure — It is not
certain from y^e Time of its Change to its Death — Its
Metamorph^s seems intended merely for y^e sake of y^e season
& multiply its kind. In y^e beginning it sheds its Clay
Coat, & spend the rest of its short Life in frisking
over y^e Waters, the Female drops her Eggs & on them
the male his Sperm on, which fall to y^e bottom are
hatched by y^e warmth of the Sun into little Worms which

Chemical Essays by Watson Professor
of Divinity - Cambridge 1781

	Lime			Matter		
	by Calcination			dissipated		
A Tun of =	11	9	16	8	9	16
Marble produced	11	1	24	8	2	4
Purbeck Stone	11	1	22	8	2	6
Clitheroe Lime stone	11	1	17	8	2	11
Chalk - - - -	11	1	9	8	2	19
Portland Stone	11	1	4	8	2	24

Several Experiments were made, in order to ascertain the utmost increase of Weight which different sorts of Lime can acquire by exposure to y. Air. The Limes were all loosely folded up in clean paper, laid in the Closet of a Room, where there was a constant Fire, & weighed at different Intervals, till they had acquired their greatest increase of Weight in most of Them attain in 20 or 30 days I have frequently observed Pieces of new burnt Lime, daily increasing at y. Rate of 10 lbs. Ton for the first 5 or 6 days. The Farmer in Limeing his Land should contrive to carry out his Lime as soon as possible after it is

burn'd, he may otherwise, for every Ton
have y^e. Trouble of carry: out 1 $\frac{1}{2}$ Ton more.

Lime which has once acquired 'tis
greatest increase, does not lose any part
of what It has acquired, by subsequent
Exposure to y^e. Air, even during the heat
of Summer: Let y^e. Matter, be what it may
attracted by y^e. Lime, 'tis a permanent Substance
Not subject to Evaporation during Summer.
Hence it cannot be doubted, that y^e. Soil
upon w^{ch}. Lime is laid, acquires a great increase
of Matter; every Ton of Lime attracts above
half a Ton of some sort of Matter, from y^e. Air
Vados It to y^e. Earth - There is great reason
to believe, that y^e. Q^{ty}. of Matter, w^{ch}. y^e. same
sort of Lime can attract from y^e. Air, depends
very much upon y^e. degree of heat, the Lime
has been burn'd; There is a certain difficult
degree of heat requisite to make y^e. Lime
attract y^e. greatest q^{ty}. fm y^e. Air. When y^e. heat
is not sufficient to convert y^e. Whole into
Lime, it must want so much of its attrac-
ting Power. Again It may want of its

Attracting Tower by Overburning, the
Stone, Vitrified. The following Experiments
cannot fail of being acceptable to y^r. reader
as they tend greatly, to corroborate what has
already been said, concerning the Nature
of calcareous Earth. We have concluded
that 200 of any calcareous Substance
consists of about 8 or 9 O. W. of some
matter or other, which is dissipated by
calcination & of between 11 & 12 O. W. of
Earth, w^{ch} cannot be dissipated in that degree
of heat — For proof of y^r. foregoing
take a Florence Flask, & pour into it
a small portion of the Acid of Sea Salt
diluted with Water, ^{then weigh it} drop into y^r. Acid
a little at a Time 20 grains of calcareous
substance, ^{pondered} & stopping gently the Top of the
Flask wth your Finger till y^r. residue be separated
then Weigh the Flask again. Now it is
obvious, that if nothing had escaped during
the solution, the weight of y^r. Flask & its
contents, after y^r. solution of the calcareous
substance, must be 20 grains more

By The following Table it appears
 What a Ton of y. Calcareous Earth
 mentioned in y. follow Table lost
 after Solution, & y. Weight of the
 Matter, or Lime remaining in y. Flask

	Lime by Solution	Matter Dissolving
	9. 16	9. 16
A Ton of Marble	12. 00. 00	8. 00. 00
Purbeck Stone	11. 00. 00	7. 3. 00
Cuthoree Limestone	13. 01. 00	6. 3. 00
Chalk, - - -	12. 3. 00	7. 1. 00
Portland Stone	12. 1. 00	7. 3. 00

It appears from y. Tryal made on several
 Substances, that pure calcareous Bodies
 lose about 9 parts in 20 of their Weight
 by being dissolved in an Acid.

When I first began making these Experiments
 I weigh'd the Flasks as soon as y. Solution
 was ended, & I found an Error on ~~losing~~ ^{Extinguishing}
 y. Quant. lost by Solution. For upon further
 Tryal I found that y. Weight of y. Flasks & its
 contents, as soon, as y. Solution was finished
 was diminished very sensibly in 3 or 4 hours
 the Cause is this, the Weight of the Air

remaining, after Solution in y^e Flask
is greater than y^e Weight of an equal bulk
of common Air. This is easily prov'd either
by suck^{ing} through a Pipe the Air contain'd
in y^e Flask, or blowing It out with a pair
of Bellows, so that y^e Flask may be fill'd
with common Air; the Flask will then be
found to weigh less y^e it did before. This
Air has a disagreeable Taste, & if suffer'd to
enter y^e Lungs Obnoxious — I took a Florence
Flask w^h would hold 26 Oz, Troy, Water, I then
pour'd In a 2 ounce Measure of the Acid of
Sea Salt, & dissolv'd slowly a q^{ty} of purest Spar
the Bottle was weigh'd as soon as y^e Solution
was ended. I let It stand for 2 hours & then blew
into It with a pair of Bellows till y^e Air became
tasteless, the Bottle was then weigh'd, It had lost
7 Grains. The Weight of Common Air filling a
Flask holding 26 Oz^{ty} of Water is about 14 gr^{ss}.
The Air w^h fill'd y^e Flask after Solution, It gain'd
Since calcareous Substances lose nearly equal
portions of their Weight, whether they are cal'd
in y^e Fire, or dissolv'd in Acids. The Matter
lost in both Cases is a Species of Air. For fast
a Glass Bladder over y^e Mouth of a Florence Flask
or other Vessel

In ^{the} Limestone is in y^e Act of Solution, the
Bladders will presently be blown up, by the Matter
w^{ch} Issues through y^e Neck of the Flask. This does
extinguishes Flame & Animal Life, - Commander
supports both. A Ton of Lime will attract half a
Ton of this Air from y^e Atmosphere ———

From various Experim^{ts} we may
conclude, that pure calcareous Substances are
composed of 9 parts in 20 of a Volatile substance
w^{ch} is dissolv^d, not only during Calcination
but during y^e Solution of y^e Substances
in an Acid & of 11 parts in 20 of an Earth
known under y^e Name of Lime

Helpe — Telleron —

The primary end of all Threatning is not Punishment, but of Prevention of it. God does not threaten that men may sin & be punished; but that they may not sin & so escape: And therefore the higher the Threatning runs of more Goodness there is in it.

— There is this Difference between Promises & Threatnings; that he who promises passes over a Right to another, & thereby stands oblig'd to him in Justice & Faithfulness to make Good his Word; but it is otherwise in Threatnings; he that threatens keeps y^e Right of Punish^g: still with him & is not oblig'd to execute what he threatened any further than y^e Reasons & End of Govern^t. require —

— Thus God absolutely threatned y^e Destruction of Nineveh; and his peevish Prophet y^e ^{understands} Threatn^g to be absolute, and was angry for being Employ'd on a Message w^{ch} was not made Good. God notwithstanding y^e Threatning he denounced, & tho' Jonah was so touch'd in Point of Honour, that he had rather have perished than ^{himself} Nineveh should have escap'd did not destroy y^e City. —

— Men are not wholly passive in believing or disbelieving, but have a great Compass of Liberty in y^e Use & Direction of those Faculties, on w^{ch} our Assent or our Perceptions depend. So far as y^e Liberty reaches they are voluntary Acts, & therefore Naturally Subject to the Law & Transgression of it. ^{as creatures} They are naturally to God. And since

The Obligations & Penalties of this Law must rise in
Proportion to its Moral Importance of its Subject
proposed to us, & no Question can be of so great Moment
as whether what is affirmed to be a Revelation of the
Will of God really is so, or not; it follows that to act
unbiased to the Laws of Reason in its Inquiry, either
to dismiss the Proposal of it without due attention,
or to suffer any Lusts, or Immoral Prejudices, to
suppress its Evidence offered for it, in even its access
of Nature, & greatest Evidences can be guilty, & in
the Exercise of these Faculties; & consequently,
such as must expose us to just Recriminations of God.
For its Plea offered in Defence of Infidelity, from
any Natural Guilt, as an Involuntary Act is false

Took 96 of Newcastle Coal & put them into an
 earthen Retort, distilled y.^m with a Fire gradually
 augmented till nothing more could be obtained
 During y.^e Distillation, there was frequent
 Occasion to give Vent to an Elastic Vapour,
 w^{ch} would otherwise have burst y.^e Vessel's employed
 in y.^e Operation. The Weights of y.^e Liquid found in
 y.^e Receiver, & of y.^e Residuum remain: in Retort
 after y.^e distillation was finished, are expressed in
 Newcastle, this Table. —

Coals 96 produced	Liquid	12	12
	Residuum	56	83½
	Loss of Weight	28	0½

We may conclude from y.^e Analyses, that Pit Coal
 contains about 8 parts by Weight in Liquid which
 consists of 3 different Substances. First, the middle
 part is an Acid Watery Fluid, of a reddish colour, it
 appears to be loaded wth Oil, & smells of a Datable
 Alkali; (2) On y.^e Surface of this Watery Fluid there
 floats a small Portion of Oil, more or less Liquid
 & transparent, accord: as y.^e Heat used in y.^e distillat.
 has been less or greater (3) At y.^e bottom there is found
 a black thick tenacious Oil, very much resembl.
 Tar. Woods distilled, that were Old & dry & cut
 into small pieces ½ inch square

96 ounces produced	Liquid	37½	61½	33½	48
	Residuum	30	26½	27½	26½
	Loss of Weight	28	8	6	21½

The Liquids separable from Woods by distillation consist partly of Oil, but principally of Water impregnated with an Acid, from w^{ch} also a volatile Alkali may be desengaged. The Oil varies in q^{ty} according to y^e Nature of y^e Wood, the hardest Woods, & hardest parts of y^e same Wood, abounding most in Oil. Oak & Box contain'd, in y^e same M^t of Wood more Oil, than Sallow. That part of y^e Oil w^{ch} is first separated from Wood, is volatile as to float upon Water, & a yellow Colour, & is q^{ty} is but small, compar'd, with that thick, black, heavy Oil w^{ch} follows. It appears that y^e Quality of y^e Liquid separable from Wood, by distillation is wholly y^e same as that from Pit Coal by the same Means — The Matter w^{ch} is lost during y^e Distillation of both Wood & Pit coal, has been call'd Air, and it certainly has One, at least, of y^e most distinguishing Properties of Air — permanent Elasticity, for Bladders may be Inflated wth It as wth common Air. It does not begin to be separated from either Wood or Coal, till y^e Lighter of y^e two Oils begins to appear. It then rushes out with great Violence, & unless a proper Vent be given the strongest Vessels will be burst by It.

On collecting this Air in Bladders, I found
that it retain'd 'till Inflammability along Time
& turn'd like y^e. Air separable from some
Metals by Solution in Acids. — From the
Researches w^{ch} have been lately made into this
Subject, It is now believ'd, that this kind of Air
may be separated fr^m all Vegetable, Animal, and
Inflammable Mineral Substances by Distillation
and the common burning of Wood, Coal, Paper, Pitch,
Oil, & other combustible Bodies, is attributed to y^e.
Air, w^{ch} being separated from y^e. Body by y^e. Heat
& Inflamm'd by y^e. Contact of y^e. Fire, continues to
burn as long as a Body affords a supply of it.
Inflammable Air, consider'd in this View, bears
a great Resemblance to what the Chemists have
hitherto understood by their Phlogiston or food of
Fire. Inflammable Air is produced by Putrefaction
as well as Distillation. Stagnant Waters, whose
Bottoms are cover'd wth putrefying Vegetables,
yield an Inflammable Air, as also in Swamps,
Gravels, Cellars & Privies — From heart of
Oak Dr. Hales got $\frac{1}{3}$ of the Weight of It of y^e.
Air. A cubic foot of dry Box Wood contains
about 5 gall^m. Liquid. Suppose the Oil & Acid
be 2 Quarts, y^e. other 18 Quarts, may be look upon
as pure Water — 'Tis a wonderfull Thing, that such
large y^e. of Water, & Air should be contain'd

with y^e Earth of Wood, by such a peculiar kind
of Union, as to remain for Ages, without losing
any of their distinguishing Properties.
The Residue remaining wth y^e Retort after the
Distillation of the Wood is a perfect Charcoal
& that w^h remains after y^e Distillation of Pit
Coal, a Cinder — *Pyracinella* an very odorous
Plant; when in full Blossom, the Air w^h surrounds
It, or a still Night may be Inflam'd by a lighted
Candle; but whether It proceeds from an
Inflammable Air exhaled by y^e Plant, or from
some of y^e finer Particles of y^e Oil of y^e Plant
being dissolved in y^e common Air of y^e Atmosphere
At Newcastle — 24 Barrels of
Coals produce generally 18 Barrels of Cinders
Cinders $\frac{1}{3}$ more in Price y^e y^e same Measure of Coals
Cambridge on y^e contrary where great Quantities
prepared for drying Malt, Coals & Cinders are y^e
same in Price & that 30 B. of y^e Coal gave 30 B.
of Cinders. I take this Difference to proceed from
y^e different Method of burning. Cambridge Cinders
Ovens are drawn Once in 24 hours. At Newcastle
only Once in 48 — The Method of making Tar
describ'd by Newmann in different parts of
Germany Norway & Sweden from y^e Pine
& Fir Trees. The Wood is inclos'd in a large Oven
w^h is of 10 or 12 Loads; This stands within another
Oven call'd y^e Mantle, the Space between

receiving the Fire. From y^e Bottom of y^e Inner
Oven runs a Gutter, by w^{ch} y^e Tarr is convey'd
off as it melt out of the Wood. This process is
a kind of Distillation, in w^{ch} y^e Inner Oven
represents y^e Body of y^e Retort, & y^e Gutter 'tis
Reck. The Fluid part of Resinous Wood are
propably melted out of Them with a less degree
of heat, than Pit Coal, Yet this Process for
procuring Tarr might be apply'd with a few
alterations to y^e obtaining the thick Oil of Coals
There would in this Plan be no loss of Fuel
The Coal in y^e Inner Oven would be converted into
y^e most Valuable kind of Linder, & the Fire
between y^e Ovens would afford a Linder as good
as y^e common Cunder — In y^e Bishoprick of
Leger y^e Coal is distilled in a kind of Still
consisting of two large Cast Iron Potts. —

In England the Coal is put into Ovens, w^{ch}
are heated by Fires lighted under their Bottoms
and Liquid Matter is forc'd through an Iron
Pipe inserted into y^e Top of y^e Oven, & w^{ch} ^{= water} communicates
with proper condensing Vessels. As y^e Pitchy Oil
is very heavy I think y^e Distill'd would be better
performed by mak^g y^e Fire on y^e Top of y^e Oven
contain^g Coal, so that y^e Liquid Matter might
run down through a Pipe inserted into y^e bottom of y^e
Oven

These Substances procured for Coal is a
corrosive Watery Liquor & two sorts of Oil.
The two Oils are put into a Iron Still to be
purify'd for y^e Watery Liquor with w^{ch} they are
mix'd — Six Barrels of y^e Oily matter
produce abt 5 Barrels of Oil of a thicker con-
sistence, Of y^e Oil thus thickend, One part is
lighter, than y^e other, the lighter part is drawn
off from y^e other, Vis not at present apply'd to any
Use, the thicker part is us'd as Tars for bottom
of Ships, but does not answer well for Cordage
— Those who are interest'd in y^e preparation
of Cook or Linder would do well to remember, that
every 96 Oz. of Coal would furnish 4 Oz. if not
6 of Oil, but if we fix it only at 5 for 100, And
suppose a Cook Oven to work off only 100 Tons
in a year of Coal, there would be a saving of 5
of Oil, which would yield A Ton of Tar: the
 requisite alterations in y^e structure of the Oven
so as to make them a kind of Distilling Vessel
might be made at a trifling Expence.

It has been Observ'd that Wood, & Pit Coal wholly
resemble each other in y^e Products w^{ch} they yield
by Distillation. And as y^e Oil from Pit Coal is found
to produce Tar, It may be worthy the Consideration
of Those who burn Wood into Charcoal to contrive
means, w^{ch} might easily be done of saving all the
Oil, w^{ch} is separated from y^e Wood on burning

3p. Nelson It may be worth While to inquire or
explain more fully the first & most obvious Notion
of Solution in ^{is} a Solid Body is united to a Fluid
- Take an Ounce of common Salt & throw it into
a Quart of Water it will fall to y.^e Bottom as Sand
but it will not stay there, in a little Time if y.^e
Water be stirred, it will uniformly be dispersed
thro y.^e whole Body of Water, no One drop of
Water will contain more particles of Salt than
another, nor will any of them contain so much
Salt as it is able to do. This power w.^{ch} the
Water has of taking up y.^e Salt, is not unlimited.
You may add so much Salt to it that it will
not dissolve One Particle more, the Water
in y.^e case is said to be saturated, all other
Menstruums are y.^e same when they will
not keep up suspended any more of y.^e Body
dissolved in them.

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— M —

Mr. Desham, computes, that Marriages one wth another
do each produce about A Births, both in England, & other
parts of Europe — Mr. King, that about One in a hund^{red}.
four Persons Marry; the Num^b. of People in England being
Estimated at ^{five} 5 Millions & half, whereof about 41,000 Ann^u.
Marry. Major Graunt from y^e London & Country Bills computes,
about 14 Males, to 13 Females. Whence he justly infers
that y^e Christian Religion prohibiting Polygamy, is more agreeable
to y^e Law of Nature, than Mahometan^{ism}. & others that allow It
And that one man ought to have but one Wife, The Surplus
of Males above Females, being spent in y^e Supplies of Wars, Seas
&c. — The Inhab^{ts}. of London are about a 10th part of y^e
whole, yet about a 5 part of y^e Revenue is paid by them.
This computed that about 7/8 of y^e People are without Property
in themselves, & for y^e heads of their Families, and forced to
work for their daily bread, & that of y^e sort there are 7 millions
in y^e whole Island of great Britain. The Subjects wthout
Property pay 3/4 of y^e Rent, & consequently enable y^e Landed
Men to pay 3/4 of their Taxes. Now if so great a part of the
Land tax were to be divided by 7 millions it would amount to
no more than 3^d for every head, therefore y^e poorest Subject
is worth 3^d to y^e Prince Penn^m.

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— Action in Oratory is a Management
of the Voice & Gesture, suited to ^{the} Matter spoken.

It is a Point well bear being controverted,
whether Action ought to be ~~continued~~ practised, or not?

A Thing w^{ch} has so much Command over Mankind,
must be Dangerous; since It is as capable of being
turn'd to our Disadvantage, as our Advantage.

— Voice & Gesture, we know, will affect Brutes;
not as They have Reason, but as They have Passions.
And our Reason & Judgm^t Itself is intended to be
by-pass'd & melin'd by Them, Action being only and as an
indirect Way of coming at Reason, where a direct and
immediate One was wanting. (viz) Where the Judgm^t
cannot be taken by the proper Means, Argument;
It is to be taken indirectly by Circumtion, & thatagen.

— Then Natural Order of Things, then, is here inverted
our Reason w^{ch} should go before, & direct our Passions
is dragg'd after Them; instead of coolly consider^{ing} & taking
Cognizance of Things, and according to what we perceive therein.
We are attacked the other Way; y^t Impression is to be carried
Backwards, by Vertue of the Natural Connection there is
between y^e Reason & Passions — The Case is much
the same here, as in Sensation & Imagination: the natural
regular Way of arriving at y^e knowl^dg of Objects, is by
Sense; an Impression begun there is propagated forward

to ϕ . Imagination, where an Image is produced similar
to that which first struck on ϕ . Organ. — But ϕ . Process
is sometimes inverted: In Hypo: Luna: & other
delirious Cases, the Image is first excited in ϕ . Imagination,
and the Impression thereof communicated back to ϕ . Organ
of Sense; by ϕ . Means, Objects are seen, ϕ . have no
Existence — To conclude Action does not tend to give
the Friend any Information about ϕ . Case in hand; is
not pretended to convey any Argument or Discourse: the
simple Use of Language would not convey, and can
any Thing help us to make a just Judgm^t. But what more may
enlarge our Understanding: When Cicero made Cato
tremble, turn pale & let fall his Paper; he did not apprise
him of any new Guilt w^{ch} Cato did not know of: the
Effect had no Dependence on Cato's understanding; nor was
it any Thing more than might have been produced by the
unmeaning Sounds of Musical Instrum^t. duly apply'd.
Logs of Timber & Stone tremble on ϕ . like Occasions

The Alkaline Salt in Maritime Plants
when exposed to y.^e heat of a glass Furnace loses in
Weight considerably but in a mild heat Nothing
hence Salt is called a fix'd Alkaly. A pound of
Common Salt contains about Half a pound
of this fix'd Alkaly

The Nitreous Acid may be obtained
without y.^e assistance of y.^e Detriolic Acid
by distilling Nitre with White Sand I
obtained a very strong fuming Acid of Nitre

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Passions

Painful Passions as well as Bodily Pain, impress the Nervous Fibres with Violent Motion, which brings them alternately into forcible contractions, Dilatations, or strengthen & increases their Muscular Force, & Action. While then this Pain or Uneasiness of Desire, annexed to y^e Passions, & impulsion of y^e Nerve is moderate & restrained within y^e Bounds of Nature, such Stimulating Desires have a good Effect, as they strengthen Muscular Motion, keep up y^e Circulation of y^e Blood, promote y^e Natural Secretions, & excite a Man to such Exercises & Actions, wherein Animal Life health & Vigour consist — But where the Uneasiness annexed to y^e ^{Passion} Desire is too violent, such a continual Stimulus will gradually derive a too great Proportion of Blood to y^e Stimulated Organs, by w^{ch} y^e Vessels will be over stretched & distended, their Muscular Force gradually impaired, & the equilibrium of y^e Blood & Juices be interrupted. And hence from a more painful Sensation, will arise a complicated train of bodily Illnesses in consequence of y^e Established Laws of y^e Union of Soul & Body — Again, while we are wearing off y^e Uneasiness of Desire annexed to any Passion, we feel a sensible Pleasure by y^e Organs fall into easy & uniform Undulations, y^e too great Conn^t of y^e Blood to them is dissolved, & y^e Equilibrium restored! And this is Circle of Animal Life as y^e Stimulus of Desire throws off y^e Indolence of Rest, & excites to Action; so Gratification moderates y^e Pain of Desire, creates a pleasant & first, & then terminates in y^e former Inaction; till just Desires return: stimulate to farther Action & continue the same Round

The acute Passions whether pleasurable or
Painfull, ~~but~~ have much of same Effects, &
operate after of same Manner as acute Diseases
do. They Effect a brisk Circulation of ^{of} Fluids, &
constrict of Solids for some ^{short} Time. Thus, sudden
gusts of Joy or Grief stimulate the Nervous Fibres,
& of Vessels of of Animal Tubes, & thereby give a greater
velocity to their included Fluids; & the Functions of
of Heart & Lungs being involuntary, they have
their more necessary & immediate Effects on them.
Thus, both sudden Joy & Grief make us breath
short, & Quick, & render of Pulse small & frequent;
though retaining our Breathe sometime to reflect
more intensely on a painfull Object, forces at
Length a strong Expiration, w^{ch} becomes a sigh.
Thus a sudden painfull Idea, making a Quick
Circulation, and thereby throwing a great Quantity
of Blood upward, makes It appear in of superficial
Vessels of of Face, Neck, & Breast, and so produces a
Blush. The same principles will account for
the Effects of Fear & Angor, w^{ch} make us change
Colour, & look red or Pale, as of Blood is accelerated
or retarded in its Course. Sudden, & great Fear do so
convulse the Nervous System, that they sometimes
alter of Disposition of of Parts, as of hair stand on

End, and the Nerves be renderd so stiff & rigid, as
to stop at Once y^e Animal Functions, whence fainting
& sometimes Death. —

Chronical passions waste y^e Nervous
System gradually. Those Nerves employ'd in contin-
ued brooding over, & fixing such a set of Ideas in y^e Imagination,
must be worn out & impair'd; and y^e ~~resulting~~ disuse, render
weak, & inactive, lifeless & destitute of sufficient Flux
of warm Blood & due Nourishment. Still & long
Grief, dark Melancholy, Hopeless Love &c. impair y^e
Habit; & sometimes when long indulg'd, terminate in
Madness; the Reason is when long indulg'd that
a constant Habit of fixing One thing in y^e Imagination,
begets a ready Disposition in y^e Nerves, to produce
again y^e same Image, till y^e Thought of It become
Spontaneous & Natural, like Breathing or y^e Motion
of y^e Heart. —

— This Spirit of Enquiry w^{ch} has been im-
planted on us by y^e Creator, and is a
necessary incitement to guide us to instruction
too often carries us beyond proper Bounds
in y^e same manner, as many other Motives
of the Soul, w^{ch} if not strong enough to
carry us to extravagant lengths, would
perhaps want sufficient Power to excite
us properly. Voltaire

The reciprocal Action of Sulphur, Iron
& Water, when properly mixed & made up
into a Paste, by its fermentation produces
a Volcano in Minature

Exp. 17. Half a pound of Flower of Brimstone
and half a pound of Steel Filings, and
1 A Ounce of Water, will, when well
mixed acquire Heat enough, to make
the Mass take Fire. — Put y^e Mixture
into an Iron Pot, cover It with a
Cloth, & Bury the whole a foot under
ground. In about 8 or 9 hours time
the Earth swelled, grew warm, & exhaled
hot sulphureous Vapours were perceived
a Flame wth delated y^e Cracks, was observed
the superincumbent Earth was covered wth
a yellow & black Powder; in short a subter-
raneous Fire producing a Volcano in Minature
If you mix y^e Oil of Turpentine wth y^e Turpentine
Acid of Nitre they will instantly take Fire &
burst forth into a dreadfull Flame — This
Expⁿ. does not always succeed wth y^e Acid
of Nitre procured from Shops, because you
seldom meet wth it strong enough, there is dan-
ger in making y^e Expⁿ. if y^e Oil is mixed even after Turpentine
I have seen a Column of Flame 20 feet high suddenly
produced by pouring a Pint of y^e Spirit on a pint of y^e Acid

1781 Doct.^r Watson's Chemical Essays
Common Salt at Cambridge
Common Salt is a neutral Salt, It
has neither an Acid nor an Alkaline taste
nor does It change the blue colour of Vegetable
into a red as Acids do, nor into a green as
Alkalies do. It consists of two Things, of an
Acid peculiar to Itself, and of ^a Alkali, wh.
is separable from y.^e Ashes of Marsh Mampire
Other maritime Plants, & wh. has been denomi-
nated, y.^e Mineral, or Fossile fixed Alkali
The Fossile Alkali, as was observ'd of the
Vegetable Alkali is more powerfully attracted
by the Acid of Vitriol, than by y.^e Acid of
common Salt; hence Common Salt is effectually
decomposed when It is distilled in conjunction
with y.^e Acid of Vitriol, for this Acid expels
the Acid of common Salt from 'tis Union
with y.^e mineral Alkali, & unites It self w.th It
in 'tis stead: the Acid of common Salt being
thus disengag'd from 'tis Basis, is easily, by y.^e
heat, rais'd in Vapour, & forc'd into y.^e Receiver
The Acid of Common Salt thus obtain'd, is

very Volatile, constantly emitting white Fumes
& It is usually called the Marine Acid, Glauber's
burning Acid, or Spirit of Salt. If y^e. reader
pours a few drops of very strong Acid of
Vitriol on a small portion of dry common Salt,
he will see a white Vapour, arising from y^e.
Salt, this Vapour is y^e most Volatile part of
y^e. Acid of the Salt, & a Dugon. may be formed
of this 'tis pungency & Volatility, from 'tis
presently infecting the Air wth. this smell is
a great content. This Acid Vapour is a
kind of Air, for It retains 'tis Elasticity for
some time, not being readily condensable by
Colo. D^r. Priestley says from his several
Experiments he finds, that this Vapour when
condens'd, constitutes an Acid, w^{ch} is twice as
heavy as air Water, the strongest Acid of Salt
consisting of $\frac{1}{3}$ of this elastic Vapour & of
 $\frac{2}{3}$ of Water. After y^e. Extraction of y^e. Acid
there is one of 'tis constituent parts of common
Salt, there remains in y^e. Vessel used for Distil^g
a compound Mass, consisting of y^e. Acid of
Vitriol united wth y^e. Mineral fix'd Alkali, the
part of common Salt.

The Salt resulting from a Decomposition of
common Salt by means of Acid of Vitriol
or from direct combination of that Acid, wth
y^e Mineral Alkali, is y^e genuine Glauber's Salt.
The Artificial composition of Common Salt
confirms y^e acco^t we have given of its constituent
parts from its Analysis: for if we combine
the Acid procured from distill^d common Salt & Acid
of Vitriol together, wth y^e Mineral fixed Alkali
there will result from their Union, a Salt in all
respects y^e same as common Salt, but somewhat
more pure - It is called regenerated Salt.

Sea Water, Brine Springs, Rock Salt
generally contain, besides Common Salt, many
various other Earthy & Saline Ingredients,
such as y^e Calcareous Earth from w^{ch} Fish shells
are probably formed; - the Earth called Magnesia
Epson's Salts, or y^e Salt resulting from y^e
combination of y^e Acid of Vitriol wth Magnesia -
Selenites; or the Salt resulting from y^e combinaⁿ
of y^e Acid of Vitriol, wth y^e Earth of y^e Nature of Fish
shells; - Glauber's Salt; - fixed Alkali wth
y^e Vinegar Acid &c. Sometimes, all these

heterogeneous Substances, and sometimes
only a few of them are found in the Water
from w^h common Salt is made; they are
all of them foreign to y^e Nature of Salt
I enquire its Quality, & hence we may
conclude, that common Salt may have
different Properties, according to y^e Quality
of the Water, or y^e Skill of y^e Salt maker
in separating these Mixtures from it

Common Salt as a Manure
So In^o Pringle has observed that common
Salt, when used in small q^{ties}, as One Ounce to
12, accelerates the Putrefaction of animal
Substances, when in larger retards It. And
hence he deduces its Utility, in assisting
the Organs of Digestion in Man, and
perhaps in other carnivorous Animals.
The same Observation may be extended to
graminivorous Animals, they are all
exceedingly fond of Salt, probably from its
great tendency to corrupt y^e Element, & to
render thereby thereby its discharge from the

It makes into y^e. Intestines more easy &
expeditions. In like manner Salt, when
apply'd as a Manure in small q^{ties}, is found
to be very beneficial; not probly from its
entering as an Aliment into y^e. Substance
of Vegetables, since there are many Expe-
riences tending to prove, that no kind of Salt can
of Itself become y^e. Food of Plants, but
from its Efficacy in reducing Weeds, dry'd
herbage, dead Roots &c. into a putrid Oily
Mafs; the fructifying Virtue of Oily Composts,
being now generally acknowledg'd, but when
It is used in large q^{ties} by preserving these
Matters from Corruption, drying up and
hardening the fibrous Capillaries of the
Roots, so that they become unfit for suck-
in y^e. Nourishment, the Fertility of y^e. Ground
is diminished, or wholly destroy'd.

Nitre or any other Salt in Iron very small
The Oils may help Vegetation; tis Salt does not
The principle Use is keeping the Earth from
Infecting & has a tendency to Putrefaction

without admitting Air, promotes
Vegetation. It appears from several Expts
that y^e solid Impurities of Rain & Snow
Water is much y^e same in Places as
far distant from each other, as Ireland
& Prussia. Marte —

As its Use in Agriculture depends on
some sort of Soils upon y^e ^{2^d} of calcareous
Earth &c. It contains, and in others upon
y^e Clay &c. enters its Composition. Some
will wish to know a Method for ascertaining
the exact proportion of Clay & Calcareous
Earth contained in any particular kind of
Marble — Take 10z of y^e prepared
Marble, previously reduced into Powder, &
well dry'd, pour upon y^e Marble a diluted Acid
of Sea Salt; an Effervescence will ensue; this
Effervescence arises from y^e Action of y^e Acid
on y^e calcareous Earth, for y^e Acid has no action
on pure Clay, nor even on Clay & Sand, as may
be seen by pouring a little of it on pipe Clay
when so much Acid has been added to y^e Marble

that no further Effervescence is observed, we
may conclude that all y^e calcareous Earth
in y^e Mael is dissolved in y^e Acid; put the
whole into a Filter y^e calcareous Earth will
pass through y^e Filter, whilst the Clay will
remain on y^e Filter. Wash y^e Clay & filtering
Paper, by pouring upon them hot Water, till
they are freed from all Saline particles; the Clay
being afterwards dry'd, as y^e Cune of Mael
had been, will shew, by its loss of Weight, the
pt^{ts} of Calcareous Earth, y^e y^e Mael contained.
As to y^e Matter w^{ch} remains upon y^e Filter, &
w^{ch} has been call'd Clay, it may chance either
be pure Clay or a mixture of Clay & Sand; If
y^e latter it may be known by washing it &
collecting y^e Sediment —

Clay not falling into powder by the Influence
of the Air, are not class'd amongst the Males
Clay, for y^e Tiles & Bricks are made often contain
a pt^{ts} of calcareous Earth, hence we may understand
y^e reason, why Bricks & Tiles cannot resist a
great degree of Moisture or Heat, & peal & roagist
fast

Bishop Watson Experiments

A Cubick foot of Wood, dry, or old, 24 Night

And in Ounces —	Box	950 Oz.
Exp. Ash in seven day	Oak	892.2
Exp. cutting lost near 3/4 of	Walnut	832.2
Surpass'd Water	Deal	615.2
If an Oak or Ash Tree	Walnut	790.2
was cut into Boards,	Mahogany	816.2

or Splinters, upon y^e. Spill felt, & saving them would be of One Ton or more besides Chips & other refuse parts — It is well known that

Wood becomes heavier y^e. Water if y^e. Air be extract from his Pores either by an Air Pump, or boiling in Water. The Woods above, were rendered heavier

than Water by long continuance in cold Water about 60 Degrees warm, they sunk in y^e. Water. Deal required 100 days soaking. Took y^e. out & let

y^e. dry a Month; Then weigh'd them — The Box Oak & Ash had each lost 32 of their Weight they

before put into Water; but Mahog^y, Walnut, & Pine had lost only 6. Most Woods contain both a gum

& resinous part. Gums are soluble & Resins not so in Water, hence y^e. Diff^t. Weight, when dry & when soaked wth. Water & the U^s. of painting wth. not only

preserves y^e. W^{ood} but preserves the Woods in a Crucible red hot for 3 hours & all converted into perfect Charcoal weigh'd warm

Walnut	259.
Oak	229.
Deal	159.
Box	209.
Ash	179.

Though Charcoal fm every sort of Wood is
incapable of being decomposed, by y^e strongest Fires
in close Vessels, yet it is a compounded Body, & may
be decomposed by burning in y^e open Air to Ashes.
This something by w^{ch} Charcoal is rendered Inflam-
mable. Why n^o Air is infected wth a particular Smell during
burning is called by Chemists the Phlogiston
but whether it be an Elastic Inflamable Fluid,
or an ^{un}elastic Earth of a particular kind, constituting
but a small part of y^e W^t of what is dispersed
in y^e Air from burning Charcoal: Wee all know.
What a strong smell is diffused thro' a large Room
from y^e Ignited Snuff of a Candle, Or fm a small p^{ty}
of Charcoal w^{ch} has not been well burnt; The Vapour
issuing from these Substances are of an Oily Nature
Very visible. The Vapour of Charcoal tho' too
subtle to be seen, may be of a nature somewhat
similar & capable of a very extensive diffusion
thro' y^e Air. An Infant has been known suddenly
to expire fm y^e smook of a Candle, blown out under
his Nose. And the Vapour of Charcoal is most
dangerous when not well burnt. It has been found
that common Atmosph^{ic} Air is a Lord in his proper
by being made to pass thro' red hot Charcoal into y^e
Vacuum of an Air pump - puts out a Candle &
Animals dye in it.

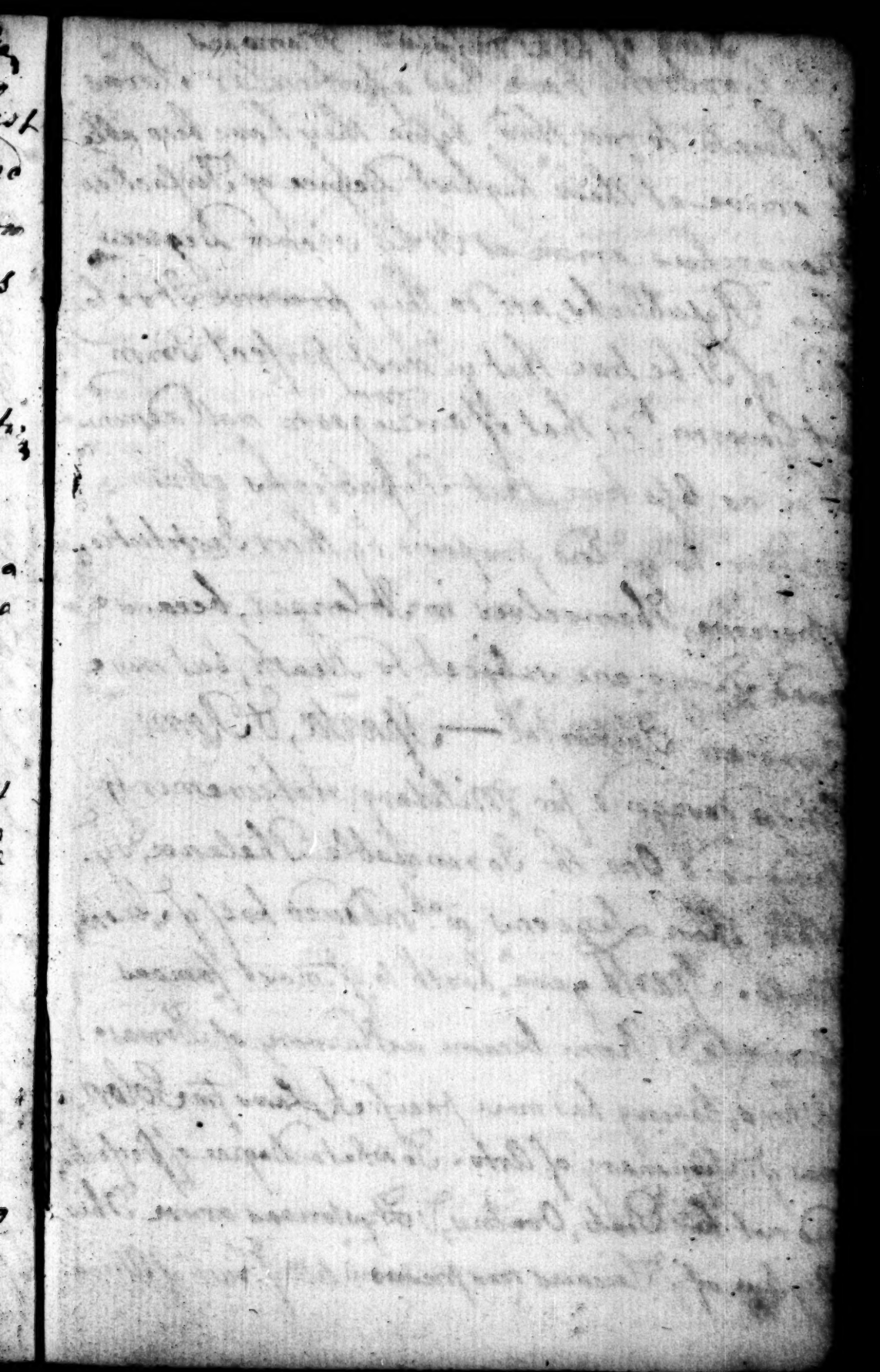
Instances of Persons who have lost their lives
burning Charcoal in an Apartment & not a
sufficient supply of fresh Air very common
in Russia. Method of Recovering, carrying
into y^e open Air & rubbing with Snow or cold
Water & pouring Water, or Milk down y^e Throat.
It is generally admitted that Charcoal & all other
Bodies cease to burn, Nitrous Gases excepted
as soon as they cease to be supplied wth fresh
Air. Exposing hot pit Coal, also Water increases
its Weight wth y^e Fire. — Animals &
Vegetables are soon reduced by putrefaction
to an Earth, many sorts of Stones & Metallick
Substances are crumbled into Dust by action
of Air & Water; but Charcoal remains unchanged
for Ages, whether exposed to Air, Immersed in Water
or buried in y^e Earth — The Beams of y^e Theatre
at Herculaneum, were converted into Charcoal
by the Lava, w^{ch} overflowed that City, & during the
Lapse of 1700 ~~years~~ Centuries, the Charcoal has
remained as entire as if it had been formed but
Yesterday, & it will probably continue so to y^e End
of the World. The famous Temple at Ephesus was
built upon Wooden Piles w^{ch} had been charred on the
Outside. Charring Ends of Posts for fixing in Earth
is very common

Kingdom of France 20 Millions
By a List in y. latter End of the last
Century there were about 90 Thousand
Persons in y. Convents of this Kingdom
Revenue abt. 50 Millions of Crowns

France of abt. 130 M. Ages
Only 80 d. w. yields good Ret.
Cash. 900 Millions Livres

People in Paris one w. another don't live
above 23 or 24 Years of
Childhood is not an Enjoyment. Life - a preparation
for w. deduct 10 y. of y. 13 y. remains
ded. half for Sleep, Inconveniences of Life
You will then have 6 1/2 y. left to pass
in vacation, in Pain, some Pleasures, &
Hopes. — At y. Rate abt. 3 1/2 y. tolerable
Existence

8 French Crowns. abt. 20 £
N.B. a Million of Livres = 43,750 Sterling
9 Thous — not quite 1000 £



History of the Prussians. Memoires
All Governm. have had a particular Series
of Events to run thro', before they have been able
to arrive at their highest Degree of Perfection.
Monarchies arrive at It by slower Degrees
than Republics, nor do they preserve It so long.
And if It be true that y. most perfect Form
of Governm. is that of a Kingdom well adminis.
It is no less true, that Republics attain
sooner to y. End propos'd by their Institution,
& preserve Themselves in It longer, because
good Things are subject to Death, but wise
Laws are Immortal — Sparta, & Rome
Cities design'd for Military achievements,
produced One the Invincible Philanax & y.
other those Legions w.^h subdued half y. known
World. Sparta gave birth to y. most famous
Generals, & Rome became a Nursery of Heroes.
Athens, having had more pacifick Laws from Solon,
was y. Seminary of Arts. To what a Degree of Perfection
did not her Poets, Orators, & Historians arrive? This
Asylum of y. Sciences was preserved till y. ruin of Attica

The foundation of the Republics of Carthage
Venice, & Holland was connected with common
This they constantly pursued & maintained, as the
principle of their Grandeur, & support of their State.
The Monarchies, & Form of Govern^t. has no other Basis
than y^e. absolute Will of a Prince, the Laws, Army
Trade & Industry are subject to y^e. caprice of a
single Man, whose successors hardly ever reason-
-able each other. Hence it generally follows that
on y^e. accession of a new Prince, y^e. State
is governed by new Principles, and this
is what hurts this Form of Govern^t.

Frailty & Instability are inseparably
connected wth y^e. Works of Man. These
Revolutions of Monarchies & Republics
have their Origin in y^e. Immutable Laws
of Nature. It is necessary y^e. human
Passions should serve as Springs for y^e.
continual shifting of new Decorations, w^{ch}.
the audacious Fury of some carries off, &
y^e. Weakness of others is incapable of
defending; that unbridled ambition should
subvert Republics, & y^e. Artifice & triumph
sometimes over Simplicity.

Were it not for these great Shocks we are speak-
of; the Universe would continue always the same
& there would be no Equality in y. Fate of Nations.
Some would be always civiliz'd, & happy, & others
always barbarous & unfortunate. We have seen
monarchies rise, & fall, and People once rude &
unpolish'd, become civiliz'd, & a model to other
Nations. May we not conclude that these
Nations have a Revolution similar to y. of the
Planets, w.^{ch} in y. Opinion of some astronomers, after
having in ten Thous. y., run thro' y. whole space
of the Heavens, find themselves, at length, at y.
very place from whence they sett out —

Luxury was introduced into Religion
upon y. Increase of Riches — Formerly the Peop^{le}
thought it improper to place their Gods in
Temples built by human Hands, for w.^{ch} reason
they worshipp'd them in sacred Groves; but
in proportion as they grew civiliz'd, their
Gods came to live on Temples, & yet y. ancient
Custom was not entirely abolish'd; for we find
that Charlemain forc'd y. Saxons to worship
Oaks, or to water them w.th y. blood of Victims

The Priests of those Days had 3 sorts
of Black Tricks: They invented Oracles
they dabbled in Astrology & Physick, to
impose upon y. Ignorant Vulgar. Hence
it was very difficult to extirpate a Religion
supported by such a Multitude of Superstitions.
All Germany was still attached to y.
worship of Idols, when Charlemagne, &
after him Henry the Fowler, undertook
to convert these People. After several
useless Efforts they succeeded only by
drowning Idolatry in Torrents of human
Blood. People may say what they will
but there is not y. least Vestige of Paganism
to be found in Brandenburgh before y. Time
of Charlemagne; or 8 Cent.th — The Impossibility
of withstanding so formidable an Enemy & the
fear of Menages, induced these People to
submit to Baptism, wth they received in y.
Emp.^{rs} Camp; but as soon as y. Danger was
over, the Emp.^r remov'd from them with his Army.
They all return'd again to their State of
Idolatry, at least they were subdued & converted.

The Idols out of their great Zeal demolished
the Pagan Idols, in so much that there are scarce
any remains of them now extant; the empty
Niches in y^e Walls of those Idols were filled with
Saints of every kind, and new Errors succeeded
to those of Antiquity, There was neither Morality
nor Learning. When Converted to Christianity
they fell into y^e very excess of false Zeal —
Brandenburg made Itself Tributary, to the
Pope, to y^e Emp^r., & to y^e Margrave's Gov^r.
But y^e People soon repented their Folly, &
regretted those Idols w^{ch} were visible Objects
of Worship, & less burthensome to them than
y^e yearly Tributes w^{ch} they p^d. to y^e Pope
whom they never saw — The Love of Liberty
the Force of Inextinguishable Prejudice, & y^e prospect
of their own Interest, led them back to their
false Gods. Mithroogus, King of y^e Vandals,
put himself at y^e head of the Pagan party, &
restored y^e ancient Worship, after driving out
the Margrave from Brandenburg — It was by force
of arms y^e Christianity was ^{re}established for the
third Time in this Country

The gross Ignorance into w^h these People
were sunk in y^e. 13 Cent: was a Soil in
which Superstition must necessarily thrive
In Fact there was no want of Miracles, nor
of any other kind of Tricks capable of estab^l
the Authority of the Priest - In y^e. 13 Cent^{ry}
most of the religious Orders were founded
The Pope established as many as he could
of them in Germany & particularly in y^e Coun^{ty}
of Brandenburg under y^e pretence of fixing
y^e Minds of y^e. People to Christianity - The Idle
the lazy, & all those who had incurred shame
& disgrace in y^e. World retired into those sacred
Asylums, where they sobd the State of kinship
by banishing themselves from Society, thus they
became a Burthen to y^e. Publick & thus with
opposition y^e. Pope raised a powerful Army of
Priests, at y^e. Expence of the several Princes, &
kept large Garrison in Count^{ies} where he had no
Sovereignty - But in those Days the People
were brutish, the Princes weak, & y^e Priests
rode in Triumph In 1351 the Plague in y^e.
Country - To appease the Divine Wrath some
Scots were baptizid by Force - Others were

Of Religion under the Reformation
by King of Prussia —

I shall not consider the Reformation as a
Divine or a Historical; the Tenets of this Relig-
ion & Events it gave rise to are so well known, that
there is no need to repeat them. So great & so
extraordinary a Revolution, w^{ch} changed
almost y^e whole System of Europe deserves
to be examined in a Philosophical Light
The Catholick Religion, w^{ch} had been rais'd
on y^e Ruins of that of the Jews, and of the
Pagans had now subsisted during the Space
of 15,00 Y^{rs}. She had been humble, & mild
under Persecution; but grown fierce after her
Establishm^t; she was for persecuting in her turn,
The Christendom was subject to y^e Pope, who was
accounted Infallible, by w^{ch} means his Power was
more extended, than that of the most Absolute
Monarch — A Petifull Fryar undertook to oppose
a Power so well established, & of a sudden, One half
of Europe shook of the Papal Yoke — As the several
Causes w^{ch} produced this great Revolution, had sub-
sisted long before It happened, they prepared the Minds
of the People for so important an Event. The Effect

Religion was degenerated to such a Degree, that
the very Characters of his Institution were no
longer discernible. Nothing could recall the
original sanctity of his Doctrine; but It was soon
perverted by y^e natural Bias of Mankind to
corruption. This Religion w^h preached Humility,
Charity & Patience was established by Fire & Sword.
The Priests who ought to have been Examples of
Purity & Poverty, led y^e most scandalous Lives - They acquired
immense Riches; w^h puffed them up with Pride, & soon
of them were become powerfull Princes - The Pope
who originally had been subject to y^e Emper^{or}, afterwards
himself the Patron of making & deposing them; he thrust
out his Excommunications, laid whole Kingdoms under
Interdicts, & carried Things to so enormous an Excess, that
y^e World was obliged to cry out for a Reformation.
(Priests married till y^e 13 Cent: Purg^{atory} was invented by
images in 1284, & transubstantiation in 1645)
The Austin Friars were in Possession of the Trade of
Indulgences; but y^e Pope gave y^e commission, y^e same
to y^e Dominicans w^h occasioned a Quarrel between
y^e 2 Orders - Luther y^e Austin exclaimed agst y^e
Pope, & attack wth great Vigour, the abuses of y^e Church
became y^e Head of a Sect, & his Doctrine spread the
Bishops of their Benefices, & y^e Monasties of their Riches.
Princes followed y^e new Reformer in Crowd.
The Reformation was of Service to

the World, & especially to y^e Progress of the
human Understanding. The Protestants being
oblig'd to reflect upon Matters of Faith, & vested
themselves suddenly of the Prejudices of Education,
found themselves at Liberty to make use of their
Reason, that Guide w^{ch} is given Man to conduct him
w^{ch} he ought to follow, if ever, in y^e most Import-
t concerns of Life. The ~~Catholics~~ Catholics find-
themselves attack'd, were oblig'd to defend them-
selves. The Clergy began to Study, & emerged from that
shamefull Ignorance, in w^{ch} they had been long
suppos'd. If there was but one Religion in y^e
World, it would be proud & despotic; The Priests
would be so many Tyrants, who, while they
exercis'd their Covinity towards y^e People, would
then Indulge only to their own Crimes —
Faith, Ambition, & Policy would enslave the
Universe. Now, that there are a great many Sects,
none of them can deviate, without having Reason
to repent it, from y^e Rules of Moderation.
The Reformation is a bridle w^{ch} hinders y^e Pope
from giving loose to his Ambition — The Catholic
& Protestant Religion Clergy, who watch one another
with an equal Inclination to criticise, are both
oblig'd at least to observe an external decorum
Thus an exact balance between them

It is as true to y. human Understanding that
at y. beginning of so learned an Age as 18th
all manner of Superstitions were yet subsisting
Men of Sense as well as y. Vulgar believed in
in Apparitions. In 1708 a Woman who had
y. Misfortune of being Old, was burnt as a
Witch — Origin of Laws It seems probable
that y. Fathers of Families were the first Legislators,
the necessity of establishing Order in their own
Houses, oblig'd them without doubt to make
Domestick Laws. After those early Ages, when
Men began to unite in Communities, the Laws
of those particular Jurisdictions were found
insufficient for a more numerous Society
The Human Heart w^{ch} seems to love its
Solitude & Malice in Solitude, exerts every branch
of it upon y. great Stage of the World. And if
the mutual Intercourse of Mankind furnish the
virtuous with good Company It supplies
accomplices also to y. Wicked — When
Disorders began to encrease in Towns & some
Vices were seen to rise — Fathers of Families
as most interested, agreed, for their own security
to make Laws, & Magistrates were appointed
to enforce them. Such is the depravity of Man
that to live happy in Peace, a constraint of Laws
is absolutely necessary.

from y^e Union of Towns. Republick took their
rise; & from y^e Bias of all human Things
to vicissitude, the Forms of their Govern^t. oftⁿ,
chang'd. The People, tired of a Democracy,
made a Transition to Aristocracy, in y^e Room
of w^{ch} they substituted, afterwards, a Monarch^y
Govern^t. — This was bro^t about two Ways;
for either y^e People plac'd their Consideration
on y^e eminent Virtues of One of their fellow
Citizens, or some ambitious Person, by artifice
usurp'd the Sovereign Power. OSIRIS is the
first Legislator mention'd in profane History
King of Egypt, & establish'd Laws for that Country
OSIRIS appointed 30 Judges he regulated the
worship of the Gods, the Division of Lands, & y^e
distinction of Ranks, & conditions: he forbade the
Persons of Debtors to be arrested. & banish'd the
seducing Charms of Rhetorick from publick
assemblies. The Egyptians pledg'd the dead
Bodies of their Fathers, & left them with their
Credit for Security, & it was almost Infamy not
to redeem them — Next to y^e Law of the
Egyptians; those of the CRETANS are the most
ancient. Their Legislator MINOS

gave out that he was Son of Jupiter,
that he had received those Laws from his
Father, in order to render them more vener-
able. Lycurgus King of Sparta made use of
Moses's Law to w^h he added some of those
of Osiris ^{as he} collected in his Travels
thro^t Egypt. He banish'd Gold, Silver, & all
sorts of Coins & superfluous Arts for his
Republick; & made an equal Division
of Land ^{among the Citizens} ~~from the State~~. As the chief
Intent of this Legislator was to form his
People to War, he discourag'd every kind
of Passion that might inervate their Courage.
With this view he permitted the promiscu-
ous Use of Women among the Citizens, by w^hch
means the State was peopled, and an Affec-
tion of soft endearm^{ts} of Marriage
was prevented. The Children were all bro^t
up at y^e Publick Expence, & when a Father
could prove his Child not sound, he was
allow'd to kill Him, a Man not able to
bear Arms is not fit to live. They eat
altogether in Publick & no difference was made
of Rank or Condition.

It is evident that whoever neglects the
Performance of their Contracts are guilty
of y^e. Breach of Faith; of a Crime that if it
should generally be imitated would dissolve
Society & throw Humane Nature into Confusion.
Continued from last Page King Lear
Strangers were forbid to make any Stage of Actors
lest their Manners sh^d. corrupt those introduced
by Lycurgus. No Punishment against Thieves
unless detected in y^e. Fact - Lycurgus aim^d
was to form a Military Republic & he
succeeded - Dracon punish'd y^e. smallest Fault
with Death, was the first who made Laws for
y^e. Athenians, but after ^{myrron} Solon, he releas'd
the Debtors & gave y^e. Citizens a power of mak^{ing}
Laws. - Men of debauch'd Morals were not
allow'd to speak in public Assemblies, he
appointed 30 Judges but afterw^{ards}. increas'd 500
They held their Sittings by Night & the
Orators were allow'd only to state y^e. Case of their
Clients without endeavouring to excite y^e. Passions
Solon made no Law ag^t. Pericles as this Crime
had never been heard of amongst y^e. Athenians, he
thought to forbid, & would give them a Notice to amend
the Athenian Laws were afterwards success^{ful}
at Rome.

It is generally acknowledged, that
Power naturally follows Property.
Therefore exorbitant Property in
Individuals must always be unfavour-
able to civil Liberty; must always
tend to produce Licentious & Faction;
because it throws exorbitant Power
into y^e Hands of Individuals: And
the greater the Inequality between the
Poor & Rich, the more the One will be
under the Influence of the other.
It should seem, then, to be the positive
Interest even of y^e most wealthy, if they
be y^e real Friends of Liberty — 'tis evident
y^e general Int^t of y^e Community; that
some legal Limitation of Property should
take Place, I speak not of y^e Probable
but y^e Expediency of such a measure.

Limitation of extended conquest & Empire
might seem an Object worthy of Attention of
the highest Powers.— Rome perished by the
avidity of unbounded Empire. Colonies when
peopled beyond a certain Degree, become a
Burthen to y^e Mother Country.— They exhaust
her Numbers; they distract her Attention;
they divide her compacted Strength.
Such extent of Colonies, when peopled
beyond a certain Degree as may be necessary
to maintain the Empire of the Seas, will
always be a just Object of British Attention
& Regard. More than this, sound Policy
perhaps could hardly dictate. This Limitation
is of more Importance, as it would naturally
set bounds to another Excess: I mean, that
of Trade & Wealth, a Demonstration of Truth
that Exorbitant Trade & Wealth are most Dangerous
to private Virtue, & therefore to public Liberty.



Every Cause hath been assigned except the
true one for y^e high Price of Provisions
It seems to be "the sinking Value of Money"
arising necessarily from the exorbitant
increase of Trade & Wealth - It follows
the Evil is incurable, excepting only by a
general Augmentation of Wages of y^e Poor

It follows that some Regulation
in Respect to Boroughs would be
of great ^{Importance} Service. For in Boroughs
contrary to all sound Policy—Poll
tax is lodged without annexed Property

The natural consequence is that
this ill placed Power will be seized
by those who are possessed of considerable
Property. Thus Power settles in the hands of
natural Education: But a Foundation

[illegible]